

KINETIC ENERGY OF TRANSLATION

The System of Study: Let us postulate a system of study consisting of a p -population of gaseous atmospheric molecules in random motion in [proximity space](#). The surface of interest is the intangible surface of an imaginary sphere surrounded by the p -population.

Definition of Kinetic Energy of Translation: Classical mechanics defines kinetic energy of translation as the energy that a body possesses while going from place to place. When evaluated, the kinetic energy of translation (u) of a body in motion is equal to one-half the product of that body's mass (m) and the square of that body's speed (v) over its path. That is,

$$u = \frac{1}{2}mv^2 \quad \text{KET01}$$

Classical thermodynamics allots $\frac{1}{2}\mathbf{k}_B\bar{T}$ of kinetic energy to each degree of freedom that a molecule possesses. $\mathbf{k}_B\bar{T}$ is called the “thermal term”. In this thermal term, \mathbf{k}_B is Boltzmann's Constant in joules per molecule per Kelvin and \bar{T} is the mean system temperature in Kelvins.

Each axis of translatory motion is entitled to one degree of freedom. In the Cartesian framework of spatial dimensions, a molecule will have three degrees of freedom over its three-dimensional true path and hence will have $\frac{3}{2}\mathbf{k}_B\bar{T}$ joules of kinetic energy of translation over that true path. However, our p -population has only a single axis of movement—the proximity axis. Therefore, it enjoys only $\frac{1}{2}\mathbf{k}_B\bar{T}$ of kinetic energy of translation.

Kinetic Energy of Translation in the p -Population: For the mean molecular kinetic energy of translation of our p -population of gaseous molecules, Equation KET01 can now be rewritten as:

$$\bar{u}_p = \frac{1}{2}\bar{m}_p\overline{v_p^2} = \frac{1}{2}\mathbf{k}_B\bar{T} \quad \text{KET02}$$

Here, \bar{u}_p is the mean kinetic energy of translation of the p -population, \bar{m}_p is that population's mean molecular mass, and $\overline{v_p^2}$ is the mean of the molecules' squared proximity speeds. This makes v_p the square-root of the mean square proximity speed (this speed is usually termed the root-mean-square proximity speed v_p^{rms}), and v_p can be replaced by sigma (σ). That replacement allows us to expand KET02 into:

$$\bar{u}_p = \frac{1}{2}\bar{m}_p\overline{v_p^2} = \frac{1}{2}\bar{m}_p\sigma^2 = \frac{1}{2}\mathbf{k}_B\bar{T} \quad \text{KET03}$$

Kinetic Energy of Translation in the *i*-Population: For the subpopulation of molecules that are actively interacting with the surface of interest, we can write:

$$\bar{u}_i = \frac{1}{2} \bar{m}_i \overline{v_i^2} = k_B \bar{T} \quad \text{KET04}$$

Since the distribution of the mean [impulse speed](#) was derived from the distribution of the mean proximity speed, the two distributions share a common value for [sigma](#). That allows us to expand KET04 into:

$$\bar{u}_i = \frac{1}{2} \bar{m}_i \overline{v_i^2} = \bar{m}_i \sigma^2 = k_B \bar{T} \quad \text{KET05}$$

Comparison of the Two Population Parameters: Comparing \bar{u}_p with \bar{u}_i gives us:

$$\bar{u}_i = 2\bar{u}_p \quad \text{KET06}$$

Derivatives for the *p*-Population: In **Molecular Speeds**, we derived distribution functions for both the *p*-configuration and the *i*-configuration of the *p*-population molecular speeds. Using those functions and relevant equations above, we can derive similar distribution functions for the kinetic energies of translation.

The derivative of KET01 above becomes

$$du = \bar{m}_i v dv \quad \text{KET07}$$

for any mode of *u* and its equivalent mode of *v*. Substitution of the proper values of *u* and *du* into the various functions for the distributions of speed and velocity will give us the distribution functions for the corresponding kinetic energies.

Distribution Function for the *p*-Population: The limited number of molecules having kinetic energies between *u_p* and *u_p+du_p* will be given by

$$dn_p = n_p \left(\frac{1}{\pi \bar{m}_i \sigma^2 u_p} \right)^{\frac{1}{2}} \exp - \frac{u_p}{\bar{m}_i \sigma^2} du_p \quad \text{KET08}$$

The mean kinetic energy of translation (\bar{u}_p) is obtained by multiplying KET08 by *u_p*, integrating over all possible values of *u_p*, and dividing by *n_p*.

$$\bar{u}_p = \sqrt{\frac{1}{\pi \bar{m}_i \sigma^2}} \int_0^{\infty} \sqrt{u_p} \exp\left(-\frac{u_p}{\bar{m}_i \sigma^2}\right) du_p \quad \text{KET09}$$

The integral, a form of the Gamma Function, has the value

$$\int_0^{\infty} \sqrt{u_p} \exp\left(-\frac{u_p}{\bar{m}_i \sigma^2}\right) du_p = \sqrt{\frac{\pi \bar{m}_i^3}{4}} \sigma^3 \quad \text{KET10}$$

so that

$$\bar{u}_p = \frac{1}{2} \bar{m}_i \sigma^2 \quad \text{KET11}$$

This is no surprise, since it is already stated in KET03.

Distribution Function for the i -Population: The distribution of $d\bar{f}_i / du_i$ is similarly derived.

Here, the distribution takes the form

$$d\bar{f}_i = \frac{\bar{f}_i}{\bar{m}_i \sigma^2} \exp\left(-\frac{u_i}{\bar{m}_i \sigma^2}\right) du_i \quad \text{KET12}$$

Here, \bar{f}_i is the mean number of [molecular interactions](#) per unit area and time.

The mean value \bar{u}_i for this expression is:

$$\bar{u}_i = \frac{1}{\bar{m}_i \sigma^2} \int_0^{\infty} u_i \exp\left(-\frac{u_i}{\bar{m}_i \sigma^2}\right) du_i \quad \text{KET13}$$

The integral has the value

$$\int_0^{\infty} u_i \exp\left(-\frac{u_i}{\bar{m}_i \sigma^2}\right) du_i = \bar{m}_i^2 \sigma^4 \quad \text{KET14}$$

so that

$$\bar{u}_i = \bar{m}_i \sigma^2 \quad \text{KET15}$$

Thus, the mean value of the impulse energy of translation (normal to the object of interest) possessed by the selected group of molecules interacting with an object of interest is exactly twice that possessed by their brethren at large—exactly as we stated in KET05.

Comparison of Kinetic Energies of Translation: A comparison of all three of our mean values of kinetic energy of translation gives us:

$$\bar{m}_i \sigma^2 = \bar{u}_i = 2\bar{u}_p \quad \text{KET16}$$

In ideal gases,

$$\mathbf{k}_B T = \bar{m}_i \sigma^2 \quad \text{KET17}$$

That is, the thermal energy of a system is equal to the kinetic energy of translation of that system. This is because ideal gases have no internal energies. In real gases, some of the thermal energy is internal, and consequently, KET17 does not always apply.

However, as we see in [The Equipartition Conundrum](#), during the vast majority of molecular collisions, both the impulse and the kinetic energy transfers are solely translational. Therefore, for the range of temperatures normally encountered in the free atmosphere, the use of Equation KET17 introduces no significant error. Hence, we will use KET17 in the equivalences that follow.

EQUIVALENCES

Algebraic manipulation of KRT03, KET05, KET16, and KET17 gives us the following equivalences:

$$\bar{u}_p = \frac{1}{2} \bar{u}_i = \frac{1}{2} \mathbf{k}_B T = \frac{1}{2} \bar{m}_i \sigma^2 = \frac{1}{2} \bar{m} \overline{v_p^2} = \frac{1}{4} \bar{m}_i \overline{v_i^2} \quad \text{KET18}$$

$$\bar{u}_i = 2\bar{u}_\sigma = 2\bar{u}_p = \mathbf{k}_B T = \bar{m}_i \sigma^2 = \bar{m} \overline{v_p^2} = \frac{1}{2} \bar{m}_i \overline{v_i^2} \quad \text{KET19}$$

Kinetic Energies of Translation in Moving Air: When the wind blows, the values of many atmospheric parameters change significantly. These changes will be functions of the wind speed and the wind direction relative to the surface of interest. Phenomena related to fluid flows are discussed in advanced papers in this collection.

Still Air Parameters: The essay [Still Air Parameters](#) gives the following values for mean kinetic energies in still air with no net evaporation or condensation. The system temperature is 25° C, and the system pressure is 1,000 hectopascals. The system vapor pressure is 10 hectopascals.

$$\bar{u}_p = \frac{1}{2} \mathbf{k}_B \bar{T} = 2.06 \times 10^{-21} \text{ joules per molecule}$$

$$\bar{u}_i = \mathbf{k}_B \bar{T} = 4.12 \times 10^{-21} \text{ joules per molecule}$$

REFERENCES

Internal References: References to other essays in this collection are linked in the essay text by hyperlinks. You may follow these hyperlinks or ignore them, as you choose.

External References: These are papers by other authors that contain statements or data that are specifically incorporated into this essay.

Arthur Brown; **Statistical Physics**; Elsevier, New York, 1970.

D. Tabor; **Gases, Liquids, and Solids**; Third Edition; Cambridge University Press, 1991.

General References: These are works that I have read carefully and whose views have helped to shape the views presented in this collection. None of these authors are have any responsibility for my many unconventional views and opinions.

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