SIGMA (σ)

The System of Study: Let us postulate a system of study consisting of a ***p***-population of gaseous atmospheric molecules in random motion in proximity space. The surface of interest is the intangible surface of an imaginary sphere surrounded by the ***p***-population. The system temperature is postulated as $\bar{T}$ and the system mean molecular impulse mass is postulated as $\bar{m}_i$.

Definition of Sigma as the Root-Mean-Square Molecular Proximity Speed: In the ***p***-population of molecular proximity speeds, ***sigma*** is the *root-mean-square* molecular speed:

$$\boldsymbol{\sigma} = \left(\frac{1}{N} \sum_{1}^{N} v_{\pm p}^{2} \right)^{\frac{1}{2}} = v^{rms} \qquad \text{SIG01}$$

Here, ***sigma*** ($\boldsymbol{\sigma}$) is measured in meters per second, N is the total number of molecules in the system, $v_{\pm p}$ is the individual molecular proximity velocity in meters per second, and v^{rms} is the square-root of the mean of the squares of the total of all N speeds.

Sigma is a pure speed. It is not a velocity. Although composed of molecular velocities ($v_{\pm p}$) prior to processing, these component velocities have their directions sheared off in the squaring process. Hence, they have no directional subscript. That is why both $\boldsymbol{\sigma}$ and v^{rms} are shown in SIG01 without subscripts.

Moreover, since the ***p***-population is in random motion, every molecular proximity velocity value in the **+*p***-direction is equaled by an equal value in the **-*p***-direction. This makes the net direction zero, which I have chosen to interpret as no directional ***p***-subscript at all. Sigma is a positive scalar term; it is not a vector term.

Definition of Sigma as the Standard Deviation of a Statistical Distribution: Another term for the *root-mean-square* of a statistical distribution is the *standard deviation*. In statistics, the standard deviation is customarily denoted by the Greek lower case letter *sigma* ($\boldsymbol{\sigma}$). I have chosen the denotation "**sigma"** because it is easier to say than either "root-mean-square value" or "standard deviation".

Definition of Sigma as a Variable in the Thermal Term: In the various equations of statistical thermodynamics, the combination $\mathbf{k_B}\bar{T}$ is ubiquitous. This common combination has been called "the thermal term". Sigma appears as an important variable in its common equivalence:

$$\mathbf{k_B}\bar{\boldsymbol{T}} = \bar{\boldsymbol{m}}_i\boldsymbol{\sigma}^2 \qquad \text{SIG02}$$

Here, $\mathbf{k_B}$ is Boltzmann's Constant in joules per molecule per Kelvin, $\bar{\boldsymbol{T}}$ is the postulated system temperature in Kelvins, and $\bar{\boldsymbol{m}}_i$ is the postulated system mean molecular impulse mass in kilograms. Note that once the system temperature and the system mean molecular mass have been postulated, sigma becomes the sole variable:

SIG02 can be rewritten as:

$$\sigma = \left(\frac{\mathrm{k_B}\bar{\boldsymbol{T}}}{\bar{\boldsymbol{m}}_i}\right)^{\frac{1}{2}} \qquad \text{SIG03}$$

The numerical value of $\boldsymbol{\sigma}$ in SIG01 is identical to its numerical value in SIG03.

Definition of Sigma in Terms of the Speed of Sound: In Speed of Sound, we see that:

$$\boldsymbol{\sigma} = \frac{\boldsymbol{v}_{sound}}{\sqrt{\gamma}} \qquad \text{SIG04}$$

Here, $\boldsymbol{\gamma}$ is the ratio of the specific heat of the system gas at constant pressure to the specific heat at constant volume.

The Role of Sigma in Probability Theory: Let us start with the probability density equation. This equation postulates a random distribution of $\boldsymbol{x}$-values in a population whose mean value is $\bar{\boldsymbol{x}}$ and whose standard deviation value is $\boldsymbol{\sigma}$:

$$\phi(\boldsymbol{x}) = \frac{\mathbf{1}}{\boldsymbol{\sigma}\sqrt{2\pi}}\exp - \frac{(\boldsymbol{x}-\bar{\boldsymbol{x}})^2}{2\boldsymbol{\sigma}^2} \qquad \text{SIG05}$$

Here, $\phi(\boldsymbol{x})$ is the probability that any element of the distribution of will have the specific value $\boldsymbol{x}$, $\bar{\boldsymbol{x}}$ is the mean value of $\boldsymbol{x}$ for the population, and $\boldsymbol{\sigma}$ is the standard deviation of the distribution. Note that the curve is defined by the two variables: the mean ($\bar{\boldsymbol{x}}$) and the standard deviation ($\boldsymbol{\sigma}$).

* * * * *

REFERENCES

Internal References: References to other essays in this collection are linked in the essay text by hyperlinks. You may follow these hyperlinks or ignore them, as you choose.

External References: These are papers by other authors that contain statements or data that are specifically incorporated into this essay.

Arthur Brown; **Statistical Physics**; Elsevier, New York, 1970.

Winitzki's Error Function Approximation, 2008.
http://homepages.physik.uni-muenchen.de/~Winitzki/erf-approx.pdf

Samuel Goldberg; **Probability – An Introduction**; Prentice Hall, Englewood, New Jersey; 1960.

Helen M. Walker and Joseph Lev; **Statistical Inference**; Holt, Rinehart, and Winston; New York, NY; 1953.

General References: These are works that I have read carefully and whose views have helped to shape the views presented in this collection. None of these authors are have any responsibility for my many unconventional views and opinions.

Arthur Brown; **Statistical Physics**; Elsevier, New York, 1970.

D. Tabor; **Gases, Liquids, and Solids**; Third Edition; Cambridge University Press, 1991.

Charles Kittel; **Thermal Physics**; John Wiley & Sons, New York, 1969.

R. R. Rogers, M. K. Yau; **A Short Course in Cloud Physics**; Third Edition; Elsevier, New York, 1989.

William D. Sellers, **Physical Climatology**; University of Chicago Press, Chicago, 1965.

