

THE BAROMETRIC FALLACY

It is often assumed that the atmospheric pressure at the surface is related to the atmospheric pressure at elevation z by a precise mathematical relationship. This relationship is that given by the barometric formula:

$$p_z = p_0 \exp\left(-\frac{mgz}{k_B T}\right) \quad \text{TBF01}$$

Here, p_z is the air pressure at elevation z , p_0 is the air pressure at base level 0 , m is the singular molecular mass, g is the singular gravitational constant, and z is the height above base level.

This form of the barometric formula is derived from the hydrostatic equation:

$$p = \rho g z = \frac{Mg}{A} \quad \text{TBF02}$$

Here, p is the pressure per unit area of surface, ρ is the singular density of the overlying column of air in a column whose base had unit area, g is the singular acceleration of gravity, and z is the effective height of the column. In the second half of the equation, M is total mass and A is total area.

The variation of pressure with height then takes the form:

$$dp = -\rho g dz \quad \text{TBF03}$$

If we assume that the atmosphere is a perfect gas under conditions of equilibrium (except for gravity, of course), then the Ideal Gas Equation of State applies:

$$p = nk_B T = nm\sigma^2 \quad \text{TBF04}$$

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Here, p is the pressure per unit area, k_B is Boltzmann's Constant in units of energy per molecule per degree Kelvin, T is the temperature of the air in degrees Kelvin, m is the singular molecular mass, and σ is the root-mean-square molecular velocity. Note carefully that m is the singular molecular mass in this Ideal Gas Equation of State, not the mean molecular impulse mass.

Now, since:

$$\rho = nm \quad \text{TBF05}$$

and:

$$nm = \frac{p}{\sigma^2} \quad \text{TBF06}$$

we can substitute for ρ and σ^2 in Equation 03, giving us:

$$dp = -\frac{pmg}{k_B T} dz \quad \text{TBF07}$$

or:

$$\frac{dp}{p} = -\frac{mg}{k_B T} dz \quad \text{TBF08}$$

If we assume that the temperature is constant throughout the atmosphere, then we can integrate equation TBF08 to obtain equation TBF01:

$$p_z = p_0 \exp\left(-\frac{mgz}{k_B T}\right) \quad \text{TBF01}$$

Please note that this equation does **not** prove that the pressure is the weight of the overlying air, since that was the basic assumption explicit in equation TBF02, our starting point. In the field of logic, proving your assumptions is considered to be bad form.

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By the way, it is the fact that at every elevation z , p has unit area that gives rise to the curious anomaly that pressure is described as the weight of an overlying *column* of air, rather than the more accurate conic or trapezoidal volume.

Members of the Flat Earth Society, rejoice!

Variations on the Barometric Formula It is interesting to note that a little algebraic simplification performed on the exponential gives us:

$$p_z = p_0 \exp\left(-\frac{gz}{\sigma^2}\right) \quad \text{TBF09}$$

and that gz and σ^2 have exactly the same dimensions, L^2T^{-2} . Since, at constant temperature, n is directly proportional to p , we can rewrite Equation 09 to read:

$$n_z = n_0 \exp\left(-\frac{gz}{\sigma^2}\right) \quad \text{TBF10}$$

This gives the number of molecules per unit volume at elevation z as a proportion of those at base level. Furthermore, since the two have the same dimensions, substituting v^2 for gz we get:

$$n_z = n_0 \exp\left(-\frac{v_{-p}^2}{\sigma^2}\right) \quad \text{TBF11}$$

This, in essence, gives us the proportion of molecules with velocity v_{-p} that would reach elevation z in a uniform gravitational field in the absence of collisions. The minus sign in the subscript simply indicates that the component velocity is *away* from the surface (our object of interest in this case).

Thus, what the barometric formula actually tells us is that the molecules of an ideal gas, under conditions of equilibrium, in a uniform gravitational field, will distribute themselves vertically in a manner that reflects the distribution of component molecular velocities. This distribution has the form of a probability

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density curve (see [The Probability Density Curve](#)), and the consequent distribution of pressure with elevation follows that form.

How well this formula can be applied to the real atmosphere depends upon how well the real atmosphere corresponds to an ideal gas under conditions of equilibrium. The answer, unfortunately, is not very well. This is because certain assumptions that are necessary for the derivation of the barometric formula do not at all describe the real atmosphere.

Problematic Assumptions in the Barometric Formula

Let us take a look at some of these assumptions.

1. The atmosphere is isothermal. It is not. Temperatures in the real atmosphere range widely. If we assume that T is surface temperature, then the fact that temperatures normally decrease with increasing elevation will lead to an overestimate of the number of molecules at elevation z , an overestimate of the frequency of impacts, and an overestimate of the mean impulse transferred per impact (see [The Nature of Atmospheric Pressures](#)). All this leads to an overestimate of pressure at elevation z .

If we take some lesser temperature, it will lead to underestimation of parameters close to the surface, and overestimation of parameters at height—albeit by lesser amounts than in the first case.

When temperature inversions exist (as they frequently do at one elevation or another) the errors can be substantial.

2. The atmosphere is uniform in molecular mass throughout. It is not. Proportions of water vapor, carbon dioxide, and ozone vary widely in the troposphere, and differences in mean molecular mass become significant in the stratosphere and ionosphere. Using a molecular mass for dry air produces significant error in the humid tropics. Ignoring real differences in molecular weight with elevation will invalidate the formula for use in the stratosphere and ionosphere.

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Moreover, the distribution inherent in Equation 11 means that there will be an isotopic differentiation with elevation. Isotopes with heavier masses will be rarer at higher elevations.

All of this completely ignores that fact that you cannot substitute the mean molecular mass for the effective molecular mass without introducing significant mathematical error (see [Molecular Masses](#)).

3. The gravitational constant is uniform. It is not. It varies with latitude, longitude, and elevations. These variations, however, are quite small and lead to only a small error.

The major flaws in applying the barometric formula to the real atmosphere, however, derive from the facts that the atmosphere is not an ideal gas and is never under conditions of equilibrium. These two factors are important enough to be treated on their own.

Atmospheric Gases as Ideal Gases: You often come across the statement that the gases of the atmosphere may be treated as ideal gases for the range of temperatures and pressures that are normally encountered in the troposphere. This statement is always given without attribution or citations.

Whether or not you consider this statement to be valid depends upon which characteristics of the atmosphere you consider important. If viscosity, conductive heat transfer, radiative heat transfer, the variability of specific heat, evaporation, condensation, ionization, and similar characteristics and processes are of little importance in what you are studying; then you may probably treat the gases of the atmosphere as ideal gases with a considerable measure of impunity.

In other words, when weather occurs, the atmospheric gases are not behaving as ideal gases. And it is weather that we are trying to explain.

However, outside of the fact that real molecular collisions with the surface are not elastic, with the molecules rebounding with either more or less kinetic energy of translation than they had prior to the collisions (heat conduction), there does not seem to be much difference in the behavior of real atmospheric gases and ideal gases in terms of understanding pressure. It is still the force exerted upon a surface by the interaction of atmospheric molecules with that surface.

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Conditions of Equilibrium: The fact that the barometric formula is valid only under conditions of equilibrium raises far more significant issues. This is because the atmosphere as a whole is never in—or even close to—a state of equilibrium. It is even questionable whether any significant portion of it is ever in a state of equilibrium.

When a parcel of air is in a state of equilibrium, the entropy of that parcel of air is maximized. No measurable changes in temperature are taking place; no measurable changes in pressure are taking place; no measurable changes in density (either number or mass) are taking place. There is no net evaporation or condensation; and no winds, currents, or other gross movements exist. In other words, there is no weather.

This, once again, raises the question of how much value is an equation that cannot be used to describe the real atmosphere. In the laboratory, using atmospheric gases in closed systems, the equation may be useful. In the free atmosphere, it has questionable value. Here's another reason why this is so.

The Bernoulli Effect: Daniel Bernoulli was the first to quantify the observation that a flow of a fluid across a surface creates a drop in pressure upon that surface. When the flow is laminar and parallel to the surface, the drop in pressure is proportional to the square of the wind velocity; i. e.:

$$\Delta p \propto -\rho w^2 \qquad \text{TBF12}$$

Where: Δp is the drop in pressure, ρ is the air mass density, and w is the laminar non-viscous wind velocity parallel to the surface. This equation is still valid, even if the surface is an imaginary one.

This means that a flow of air at any elevation in the atmosphere creates a drop in pressure on both the underlying and the overlying air. It is this pressure drop that leads to the phenomenon generally known as “entrainment”. This occurs when moving air undergoes a net gain in molecular number density due to the fact that more molecules are entering the moving stream of air than are leaving it.

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Moreover, the pressure drop due to air flow is cumulative; that is, a westward flow at one elevation does not cancel out an eastward flow at a different elevation. Instead, the two (or more) pressure drops are additive. If these winds persist for any appreciable length of time, differential rates of molecular diffusion will transmit the pressure change to other elevations, and—eventually—to the surface.

The effects of vertical movements of air on pressure are even more dramatic. Subsidence can create substantial increases in pressure, and updrafts can create substantial decreases in pressure. These are not predicted by the barometric formula.

Since, at some elevation or other, winds are almost always blowing; and updrafts and downdrafts are the normal order of atmospheric business this means that any attempt to use the barometric formula to predict the pressure at elevation z from the pressure at some other elevation is almost certain to be in error.

Necessity for Recalibration: It should be noted that the International Standard Atmosphere (ISA) is based on dividing the atmosphere into six independent layers and having different parameter values for the barometric formula for each layer. Even then, they completely ignore humidity.

Agreement with Observations: Finally, we should note that when observations are taken via various means, the values for the *observed* pressures do not correspond to the projected values very closely.

This is to be expected, of course. Whenever we take a formula that is valid only for an ideal gas under conditions of equilibrium and apply it to the real atmosphere, we should not be surprised if it doesn't work very well.

Summary: The barometric formula is of very limited use in predicting actual pressures at one elevation from pressures at another elevation even in still air with no weather occurring. When the wind is blowing and weather is occurring, such predictions are almost valueless.

REFERENCES

INTERNAL REFERENCES: These are other papers in this collection that are cited or linked during the course of the discussion.

[The Nature of Atmospheric Pressures](#) – This paper defines gas pressures in terms of kinetic gas theory and statistical mechanics.

[Molecular Masses](#) – This paper shows how the values for the various atmospheric molecular masses are calculated.

[The Probability Density Curve](#) – This paper gives a brief treatment of the history and utility of the probability density curve.

EXTERNAL REFERENCES: These are papers by other authors that contain specific statements or bits of data that are specifically incorporated in the above discussion. Specifically, these papers treat gaseous flows from the standpoints of kinetic gas theory and statistical mechanical theory.

D. Tabor; **Gases, Liquids, and Solids**; Third Edition; Cambridge University Press, 1991.

Arthur Brown; **Statistical Physics**; Elsevier, New York, 1970.

James Jeans; **An Introduction to the Kinetic Theory of Gases**; Cambridge Library Collection, 1940.

GENERAL REFERENCES: These are papers by other authors that contain general treatments of kinetic gas theory, statistical mechanics and thermodynamics, atmospheric physics, and other scientific fields that are used in the above discussion.

Charles Kittel; **Thermal Physics**; John Wiley & Sons, New York, 1969.

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William D. Sellers, **Physical Climatology**; University of Chicago Press, Chicago, 1965.

R. G. Barry and R. J. Chorley; **Atmosphere, Weather, and Climate**; Holt, Rinehart and Winston; New York, 1970.

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