THE SPEED OF SOUND

The speed of sound in atmospheric air is one of those parameters whose calculated value (based on the parameters of an ideal gas under conditions of equilibrium) matches the measured value so closely that scientists customarily use that calculated value in preference to a measured value with a high level of confidence.

There are quite a few ideal gas formulae for the speed of sound. Let's start with two of the most common—both applying under conditions of equilibrium:

$$\boldsymbol{v_s} = \left(\gamma \frac{\mathbf{k_B} \bar{\boldsymbol{T}}}{\boldsymbol{m}} \right)^{\frac{1}{2}} \qquad \text{SOS01}$$

and

$$\boldsymbol{v_s} = \left(\gamma \frac{\bar{\boldsymbol{p}}}{\bar{\rho}} \right)^{\frac{1}{2}} \qquad \text{SOS02}$$

Here, $\boldsymbol{v_s}$ is the speed of sound, $\boldsymbol{\gamma}$ is the ratio of the specific heat of the gas at constant pressure to the specific heat of the gas at constant volume, $\mathbf{k_B}$ is Boltzmann's Constant in units of energy per degree per molecule, $\bar{\boldsymbol{T}}$ is the mean gas temperature in degrees Kelvin, $\boldsymbol{m}$ is the singular mass of the gas, $\bar{\boldsymbol{p}}$ is the undisturbed mean pressure of the gas, and $\bar{\boldsymbol{\rho}}$ is the undisturbed mean density of the gas.

In an ideal gas,

$$\mathbf{k_B}\bar{T} = m\boldsymbol{\sigma}^2 \qquad \text{SOS03}$$

Here, $\boldsymbol{\sigma}^2$ is the mean of the squares of the component velocity along any single axis or combination of such axes.

Ideal gas density is defined as

$$\bar{\rho} = \bar{n}m \qquad \text{SOS04}$$

Algebraic substitution and manipulation reduces both SOS01 and SOS02 to

$$v_s = (\gamma)^{\frac{1}{2}}\,\boldsymbol{\sigma} \qquad \text{SOS05}$$

Since all of our molecular speeds and velocities are expressed in terms of $\boldsymbol{\sigma}$, this means that the speed of sound in an ideal gas is a function of some sort of molecular velocity.

Molecular Speeds and Velocities gives the value for the mean impulse speed, $\bar{v}_i$, as

$$\bar{v}_i = \left(\frac{\pi}{2}\right)^{\frac{1}{2}}\boldsymbol{\sigma} \qquad \text{SOS06}$$

or

$$\bar{v}_i = \mathbf{1.2533}\;\boldsymbol{\sigma} \qquad \text{SOS07}$$

The value of gamma ($\boldsymbol{\gamma}$) in dry air is generally given as 1.403. This gives us a coefficient of

$$v_s = (\gamma)^{\frac{1}{2}}\,\boldsymbol{\sigma} = \mathbf{1.1845}\;\boldsymbol{\sigma} \qquad \text{SOS08}$$

Comparing the gamma-derived speed of sound with the mean impulse speed gives us the ratio

$$\frac{\bar{v}_i}{v_s} = \frac{1.2533}{1.1845} = 1.0581 \qquad \text{SOS09}$$

This is certainly suggestive, if not absolutely convincing. It suggests that some aspect of the distribution function of molecular impulse speeds is responsible for the transmission of sound waves in the atmosphere. It is obvious that some sort of molecular movement must carry the sound waves, and no other mean molecular speed comes as close.

The mode of this impulse speed distribution is the most likely candidate, since the wavelength of sound is measured by the distance between the modes of the sound waves. If we accept a value for the mode of

$$\widehat{v}_i = \frac{2\sqrt{\pi}}{3}\sigma = 1.1816\ \sigma \qquad \text{SOS10}$$

This is as close to SOS08 as we are likely to get in this imperfect world of ours.

Evaluation of Sigma (σ)

Equation SOS05 can be rewritten to read,

$$\sigma = \frac{v_s}{(\gamma)^{\frac{1}{2}}} \qquad \text{SOS11}$$

At NTP, the speed of sound in dry still air has been measured at 331.55 meters per second. The value of *gamma* has been measured at 1.403. Substituting these values into SOS11 gives us a value for *sigma* of 279.91 meters per second.

Substitution of this value into SOS10 gives us a value for the mode of the impulse velocity of dry still air at NTP of 330.75 meters per second.

As the saying goes, “close enough for government work”.

Speed of Sound in Moving Air: When the wind is blowing, the distribution of molecular speeds and velocities changes significantly from the still air distributions. Consequently, the means and modes of these distributions change as well. Basically, what happens is that the speed of sound increases by the wind velocity in the windward direction and decreases by the wind velocity in the leeward direction.

The relationships are extremely complex, and will be discussed in future papers.

TABLE SOS01

SPEED OF SOUND IN DRY AIR AT SELECTED TEMPERATURES

SYSTEM TEMP	SYSTEM TEMP	MEAN IMPULSE VELOCITY	SPEED OF SOUND	MODAL IMPULSE VELOCITY
°C	K	$\bar{v}_i$	$\bar{v}_s$	$\widehat{v}_i$
		m sec^{-1}	m sec^{-1}	m sec^{-1}
35	308.15	373.32	351.96	351.96
30	303.15	370.28	349.08	349.10
25	298.15	367.22	346.18	346.21
20	293.15	364.12	343.26	343.29
15	288.15	361.01	340.31	340.35
10	283.15	357.86	337.33	337.38
5	278.15	354.69	334.33	334.39
0	273.15	351.48	331.30	331.37
-5	268.15	348.25	328.24	328.33
-10	263.15	344.99	325.16	325.25
-15	258.15	341.70	322.04	322.15
-20	253.15	338.37	318.89	319.01
-25	248.15	335.01	315.72	315.84

REFERENCES

INTERNAL REFERENCES: These are other papers in this collection that are either cited or linked during the course of the discussion or whose content is especially relevant to the current discussion.

Molecular Speeds and Velocities – This absolutely essential paper defines the various velocity terms ($\boldsymbol{\sigma}, \bar{\boldsymbol{v}}_p, \bar{\boldsymbol{v}}_i$) used throughout this collection of papers and shows how they are derived and how they relate to one another mathematically.

EXTERNAL REFERENCES: These are papers by other authors that contain statements or data that are specifically incorporated into the current discussion.

Wikipedia: **Speed of Sound**; retrieved on 30 June 2011.
http://en.wikipedia.org/wiki/Speed_of_sound#Practical_formula_for_dry_air

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Computational System: All calculations were carried out by Microsoft's© Excel Program 2003 (SP3). The reader should be alerted to the fact that this computational system uses fifteen significant figures for all calculations—whether or not such precision is warranted by the accuracy of the data.