# WIND-INDUCED TEMPERATURE CHANGES 

## Introduction

In this paper, we do not treat air temperature as an intrinsic property of the mixture of gases that make up our atmosphere. Instead, we consider it to be a function of the scale of both the place and the time, and also a function of the relationship between the air and the sensor.

In dealing with parcels of air, atmospheric scientists are accustomed to the fact that the temperatures sensed in one part of the parcel will be different from those sensed in a different location. Moreover, we normally expect that these same temperatures will change significantly with time as the day or night progresses. That is why our symbol for temperature $(\overline{\boldsymbol{T}})$ has a bar over it. This bar denotes that the term is the mean of several values, and not a single individual value.

We also recognize that the temperature sensed by a sensor will have one value when both the air and the sensor are still, other values (depending upon wind direction and velocity relative to the sensor) when the air is moving, a third set of temperature values when the sensor is moving in still air, and a fourth set of values when the sensor is moving in moving air.

No. Air temperatures are not simple things.

## The Universal Gas Temperature Equation

In The Nature of Atmospheric Temperatures, we developed a Universal Gas Temperature Equation. This equation defined atmospheric temperatures as:

$$
\begin{equation*}
\overline{\boldsymbol{T}}=\frac{\overline{\boldsymbol{u}}_{i}}{\mathbf{k}_{\mathrm{B}}} \tag{WIT01}
\end{equation*}
$$

Here, $\overline{\boldsymbol{T}}$ is the mean temperature of the air in Kelvins and $\overline{\boldsymbol{u}}_{\boldsymbol{i}}$ is the mean kinetic energy of translation in joules of that sub-population of molecules that actually impact on the temperature sensing surface during some interval of time. These impacts and their resulting impulses are measured normal to the sensing surface at the instant of impact. The constant of proportionality is $\mathbf{k}_{\mathbf{B}}$, Boltzmann's Constant in joules per molecule per Kelvin.

This equation is both simple and elegant. It is simple in that it relates a single thermodynamic measurement (temperature) to a single statistical mechanical parameter (mean axial arm kinetic energy of translation). It is elegant in that it fulfills all three of our Three Principles.

The temperature of our parcel of air is solely a response to those molecules that interacted with the sensing surface (First Principle). The kinetic energy of that select sub-population of interacting molecules differs from the kinetic energy of the general population (Second Principle). Only the kinetic energy of translation normal to the surface enters into the equation (Third Principle).

Being essentially a definition, this universal temperature equation is valid for ideal gases and for real gases, under conditions of equilibrium and under conditions of non-equilibrium, in the laboratory and in the free atmosphere, for still air and for moving air. It is even valid for temperatures sensed by moving sensors in moving air.

## The Nature of Wind

In The Nature of Wind, we saw that the number of molecules with a component of movement along each of the six axial arms of an orthogonal tri-axial reference system changed when the wind was blowing from what these number were in still air. In addition, the mean axial arm molecular speeds changed as well.

When the air is still, the number of molecules with a component of motion along each of the axial arms is the same. Moreover, the mean axial speeds are equal. When the wind is blowing, these values change.

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However, these changes are strictly related to the reference system being used. If the reference system moves along with the wind, then no change can be perceived.

Put another way, when the wind is blowing, a stationary observer will see changes in the number of molecules having a component of motion along each of the six axial arms compared those numbers in still air. In addition, this observer will see the molecules moving with different mean speeds along the various axial arms compared to those speeds in still air. An observer moving with the wind will perceive no difference in the two populations, and will view the air as if it were completely still-as, indeed, it is (as far as the moving reference system is concerned).

## Limitations of the One-Dimensional Notation System

In previous papers, we used a one-dimensional notation system with $\boldsymbol{p}$ and $\boldsymbol{i}$ subscripts to denote molecular directions of movement relative to the object of interest. We need a second notation system to denote wind direction relative to our object of interest, which in this paper will be the sensing surface of a thermometric sensor.

We developed such a system, using superscript arrows to denote wind direction, in The Nature of Wind. In this paper, we will be drawing heavily on the concepts developed in that paper.

Wind System Parameters: In our discussion of winds, we will make use of the following parameters. Unless these terms bear a directional superscript or subscript as defined below, they are solely scalar terms. When they do bear such a superscript or subscript, the parent terms are the scalar component of a vector expression.
$\overline{\boldsymbol{\omega}} \quad$ Mean wind speed along the wind axis in meters per second.
$\boldsymbol{a} \quad$ Error function of the mean wind speed expressed in standard deviations: $\boldsymbol{a}=\boldsymbol{e r f}(\overline{\boldsymbol{\omega}} / \boldsymbol{\sigma})$. Note that $\boldsymbol{a}+\boldsymbol{b}=\mathbf{0 . 5}$.
$\boldsymbol{b} \quad$ Complimentary error function of the mean wind speed expressed in standard deviations: $\boldsymbol{b}=\boldsymbol{\operatorname { e r f f }}(\overline{\boldsymbol{\omega}} / \boldsymbol{\sigma})$. Note that $\boldsymbol{a}+\boldsymbol{b}=\mathbf{0 . 5}$.
$\boldsymbol{\alpha} \quad$ Angle of incidence of the wind vector to the sensing surface in degrees. When the wind is normal to and toward that surface, $\boldsymbol{\alpha}$ is $+90^{\circ}$. When the wind is parallel to that surface, $\boldsymbol{\alpha}$ is $0^{\circ}$. When the wind is normal to and away from that surface, $\boldsymbol{\alpha}$ is $-90^{\circ}$. Intermediate angles of incidence will have intermediate values of $\boldsymbol{\alpha}$. These values will range from $+90^{\circ}$ through $0^{\circ}$ to $-90^{\circ}$.
$\overline{\boldsymbol{T}}_{\boldsymbol{i}} \quad$ Mean temperature sensed by the sensing surface, measured normal to the sensing surface in Kelvins.
$\overline{\boldsymbol{p}}_{i} \quad$ Mean pressure sensed by the sensing surface, measured normal to the sensor in Pascals.
$\overline{\boldsymbol{n}} \quad$ Mean molecular number density in number of molecules per cubic meter.
$\overline{\boldsymbol{m}}_{\boldsymbol{i}} \quad$ Mean molecular impulse mass in kilograms. This is not the same as the mean molecular mass of the total population of molecules. See Molecular Masses for the difference.
$\overline{\boldsymbol{u}}_{\boldsymbol{p}} \quad$ Mean component molecular kinetic energy of translation in joules, measured normal to and toward the sensor along a single axial arm.
$\Sigma \boldsymbol{u}_{\boldsymbol{p}} \quad$ The sum of those molecular kinetic energies of translation in joules for a particular axial arm-in this case, the $\boldsymbol{p}$ arm.
$\overline{\boldsymbol{v}} \quad$ Mean molecular speed in meters per second. Subscripts will indicate whether the term refers to the general population $(\boldsymbol{p})$ or the interactive sub-population (i).
$\mathbf{k}_{\mathrm{B}} \quad$ Boltzmann's Constant in joules per molecule per Kelvin.
$\bar{f}_{i} \quad$ Mean molecular flux in number of molecules per square meter per second, measured normal to and toward the sensor.
$\overline{\boldsymbol{\varphi}}_{i} \quad$ Mean molecular impulse per impact in joules, measured normal to and toward the sensor.
$\sigma$
Sigma. This is the axial root-mean-square molecular speed in meters per second in still air at temperature $\overline{\boldsymbol{T}}$ and mean molecular impulse mass $\overline{\boldsymbol{m}}_{\boldsymbol{i}}$. As such, it is also the standard deviation $(\boldsymbol{\sigma})$ of the axial molecular speed distribution in still air. As a speed and not a velocity, it is a scalar term, possessing only magnitude.

## Superscripts and subscripts: The following terms may be used as either subscripts or as superscripts, depending upon which position is available.

p Denotes membership in that portion of the general population of molecules that has a component of movement normal to and toward the object of interest.
$\boldsymbol{i} \quad$ Denotes membership in that portion of the interactive sub-population of molecules whose next or current interaction is with the object of interest.
$\rightarrow \quad \begin{aligned} & \text { Denotes that the molecules designated by the parent expression have a } \\ & \text { component of motion in the windward direction. }\end{aligned}$
$\leftarrow \quad \begin{aligned} & \text { Denotes that the molecules designated by the parent expression have a } \\ & \text { component of motion in the leeward direction. }\end{aligned}$
$\downarrow \quad \begin{aligned} & \text { Denotes that the molecules designated by the parent expression have a } \\ & \text { component of motion normal to and toward the wind axis. }\end{aligned}$

## The Nature of Wind

We were able to capture temperature with a single equation, WIT01. Wind is a far more elusive quarry. From the standpoint of kinetic gas theory and statistical mechanics, we do not, as yet, have a single equation that captures both

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the wind direction and its force or magnitude. The Nature of Wind gives us several such equations, depending upon the angle of incidence.

Nevertheless, let us continue our discussion of wind induced temperature changes by defining once more just exactly what we mean by wind:

When a wind is blowing, a stationary reference system will measure more wind molecules with a component of movement in the windward direction than molecules having a component of movement in any other direction. In addition, this system will measure the windward molecules as moving with a higher mean speed than the mean molecular component speed in any other direction.

A reference system moving with the wind will measure the same molecular parameters as it would in still air at the same temperature, pressure, and molecular number density. The terms and expressions in the equation of state:

$$
\begin{equation*}
\bar{p}_{i}=\bar{n} \mathbf{k}_{\mathrm{B}} \overline{\boldsymbol{T}}=\bar{n} \bar{m}_{i} \sigma^{2} \tag{WIT02}
\end{equation*}
$$

have exactly the same numerical values in both still air and when the wind is blowing - as long as the sensors are moving with the wind.

Technically speaking, the above equations apply only to ideal gases under conditions of equilibrium. However, these equations are accurate when applied to the mixture of gases we call the atmosphere, under the normal range of conditions found in the free atmosphere and to within the limits of generally accepted scientific precision - that is, the proper order of magnitude and three significant figures.

As we will see below, the numerical value of the temperature parameter ( $\overline{\boldsymbol{T}})$ will change significantly when measured relative to a stationary reference system when the wind blows.

## Wind Induced Temperature Changes

When the wind blows, both the pressures and the temperatures sensed by a sensing surface exposed to the wind will change. The magnitude of these changes
will depend upon the wind speed and the angle of incidence that the wind makes with the sensing surface.

The resulting changes in pressure are dealt with in the paper, Wind Induced Pressures. This paper attempts to deal with the resulting changes in temperatures. It should be noted at this point that these changes are quite small at normal wind speeds.

Let us postulate a wind blowing upon an imaginary thermometric surface. Table 4 in The Nature of Wind gives us the following increment to the static temperature upon such a windward sensor:

$$
\begin{equation*}
\Delta \overline{\boldsymbol{T}}=\frac{\Delta \bar{u}_{i}}{\mathbf{k}_{\mathrm{B}}}=\frac{\bar{m}_{i} \bar{v}_{p} \vec{v}_{i}-\overline{\boldsymbol{m}}_{i} \bar{v}_{p} \bar{v}_{i}}{\mathrm{k}_{\mathrm{B}}} \tag{WIT03}
\end{equation*}
$$

Here, $\Delta \overline{\boldsymbol{T}}$ is the change in the air temperature sensed by the sensor when the wind begins to blow. The right-hand expression denotes the wind-induced temperature minus the still-air temperature. The reader is directed to the original document for an elaboration of and the derivation of the individual parameters.

## Mean Windward and Leeward Molecular Kinetic Energy of Translation -

The mean molecular kinetic energy of translation in the windward direction of that sub-population of molecules having a component of movement in the windward direction is given by:

$$
\begin{equation*}
\bar{u}_{p}^{\vec{n}}=\bar{u}_{p}+\Delta \bar{u}_{p}^{\overrightarrow{ }}=\bar{u}_{p}+\frac{1}{2} \overline{\boldsymbol{m}}_{i} \overline{\boldsymbol{\omega}}^{2} \tag{TMA05}
\end{equation*}
$$

The mean molecular kinetic energy of translation in the leeward direction of that sub-population having a component of movement in the leeward direction is given by:

$$
\bar{u}_{-p}^{\leftarrow}=\bar{u}_{p}+\Delta \bar{u}_{p}^{\leftarrow}=\bar{u}_{p}-\left(\frac{1}{2} \overline{\boldsymbol{m}}_{i} \bar{\omega}^{2}\right)
$$

TMA06

Redefining the Thermal Equation - The next step is to take the thermal equation (TMA01) and redefine it in hopes of finding a term in common with TMA05 and TMA06. To do this we use Equation KET22 from Kinetic Energies of Translation:

$$
\begin{equation*}
\overline{\boldsymbol{T}}=\frac{2 \overline{\boldsymbol{u}}_{p}}{\mathbf{k}_{\mathbf{B}}} \tag{TMA07}
\end{equation*}
$$

Temperatures Sensed by a Windward Sensing Surface - Combining Equations TMA05 and TMA07 gives us the temperature sensed by a windward sensing surface normal to the wind:

$$
\begin{equation*}
\bar{T}^{\rightarrow}=\bar{T}+\frac{\mathbf{2 \Delta} \bar{u}_{p}}{\mathbf{k}_{\mathrm{B}}}=\overline{\boldsymbol{T}}+\frac{\bar{m}_{i} \bar{\omega}^{2}}{\mathbf{k}_{\mathrm{B}}} \tag{TMA08}
\end{equation*}
$$

Temperatures Sensed by a Leeward Sensing Surface - Combining Equations TMA06 and TMA07 gives us the temperature sensed by a leeward sensing surface normal to the wind:

$$
\begin{equation*}
\bar{T} \leftarrow=\bar{T}+\frac{\mathbf{2 \Delta} \bar{u}_{-p}^{\leftarrow}}{\mathbf{k}_{\mathrm{B}}}=\bar{T}-\frac{\bar{m}_{i} \bar{\omega}^{2}}{\mathbf{k}_{\mathrm{B}}} \tag{TMA09}
\end{equation*}
$$

## Kinetic Energy Budgets in Moving Air

At this point, it is useful to step away from a focus on temperatures and to return briefly to dealing with kinetic energies of translation. Let us look more closely at how these energies change when the wind starts to blow.

Windward Kinetic Energy Increase - We start with the increase in kinetic energy of translation along the windward arm of that sub-population of molecules ( $\boldsymbol{n} \boldsymbol{)}$ that has a component of motion in that windward direction. The number of molecules in this sub-population is:

$$
\begin{equation*}
n^{\rightarrow}=\bar{n}\left(1-e r f c\left(\frac{\bar{\sigma}}{\sigma}\right)\right) \tag{TMA10}
\end{equation*}
$$

When the wind blows, the mean kinetic energy of translation of these molecules along the windward arm changes by:

$$
\begin{equation*}
\Delta \bar{u}^{\rightarrow}=+\left(\frac{1}{2} \bar{m}_{i} \bar{\omega}^{2}\right) \tag{TMA11}
\end{equation*}
$$

Thus, the change in the total kinetic energy in the windward direction becomes:

$$
\begin{equation*}
\Delta u^{\rightarrow}=+\left(n \rightarrow \vec{u}^{\rightarrow}\right)=+\left(\bar{n}\left(1-\operatorname{erfc}\left(\frac{\bar{\sigma}}{\sigma}\right)\right)\right)\left(\frac{1}{2} \bar{m}_{i} \bar{\omega}^{2}\right) \tag{TMA12}
\end{equation*}
$$

Leeward Kinetic Energy Decrease - We continue with the decrease in kinetic energy of translation along the leeward arm of that sub-population of molecules ( $\boldsymbol{n}$ ) that has a component of motion in that leeward direction:

$$
\begin{equation*}
n^{\leftarrow}=\bar{n}\left(\operatorname{erfc}\left(\frac{\bar{\sigma}}{\sigma}\right)\right) \tag{TMA13}
\end{equation*}
$$

When the wind blows, the mean kinetic energy of translation of these molecules along the leeward arm changes by:

$$
\begin{equation*}
\Delta u^{\leftarrow}=-\left(\frac{1}{2} \bar{m}_{i} \bar{\omega}^{2}\right) \tag{TMA14}
\end{equation*}
$$

Thus, the change in the total kinetic energy in the leeward direction becomes:

$$
\begin{equation*}
\Delta u^{\leftarrow}=-\left(n^{\leftarrow} u^{\leftarrow}\right)=-\left(\bar{n}\left(\operatorname{erfc}\left(\frac{\bar{\omega}}{\sigma}\right)\right)\right)\left(\frac{1}{2} \bar{m}_{i} \bar{\omega}^{2}\right) \tag{TMA15}
\end{equation*}
$$

Change in the Axial Kinetic Energy - The net change along the windward/leeward axis thus becomes:

$$
\begin{equation*}
\Delta u^{\rightleftarrows}=\left(n^{\rightarrow} \overrightarrow{u^{\prime}}\right)-\left(n^{\leftarrow} u^{\leftarrow}\right) \tag{TMA16}
\end{equation*}
$$

This can be expanded to:

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$$
\begin{equation*}
\Delta u^{\rightleftarrows}=\left[\left(\bar{n}\left(1-\operatorname{erfc}\left(\frac{\bar{\sigma}}{\sigma}\right)\right)\right)-\left(\bar{n}\left(\operatorname{erfc}\left(\frac{\bar{\sigma}}{\sigma}\right)\right)\right)\right]\left[\frac{1}{2} \bar{m}_{i} \bar{\omega}^{2}\right] \tag{TMA17}
\end{equation*}
$$

This expression can be simplified to read:

$$
\begin{equation*}
\Delta u^{\rightleftarrows}=2\left(\bar{n}\left(\operatorname{erf}\left(\frac{\bar{\sigma}}{\sigma}\right)\right)\right)\left(\frac{1}{2} \bar{m}_{i} \bar{\omega}^{2}\right) \tag{TMA18}
\end{equation*}
$$

This simplifies even further into:

$$
\begin{equation*}
\Delta u^{\rightleftarrows}=\left(\operatorname{erf}\left(\frac{\bar{\sigma}}{\sigma}\right)\right)\left(\bar{n} \bar{m}_{i} \bar{\omega}^{2}\right) \tag{TMA19}
\end{equation*}
$$

When the wind blows, the kinetic energy along the wind axis increases by an amount that is proportional to the square of the wind velocity-as Bernoulli pointed out several centuries ago.

The question now becomes, where does this energy come from? The observer drifting with the wind senses no increase in mean kinetic energy in his enveloping parcel of air, so the increase must come from internal sources. The obvious sources (which Bernoulli anticipated) are the kinetic energies of the other four axial arms of movement. When the wind blows, each of these energies is diminished by a quarter of the amount in TMA19. This has predictable consequences on the temperatures sensed by sensing surfaces normal to those directions; i. e., by a sensing surface parallel to the wind direction.

## Molecular Parameters on a Surface Parallel to the Wind

At this point, we can abandon our normal curve and the sub-populations under it. It is only useful when our imaginary reference plane is normal to the windward-leeward axis. Let us now turn that imaginary reference plane until it is parallel to the wind direction. Later, we will see what happens at intermediate angles.

When the air is still, the number of molecules having a component of movement normal to and toward a plane parallel to the future wind direction is $\frac{1}{2} \overline{\boldsymbol{n}}$. This number does not change when the wind starts to blow. Therefore,

$$
\begin{equation*}
n^{\downarrow}=\frac{1}{2} \bar{n} \tag{TMA20}
\end{equation*}
$$

When the air is still, the mean component kinetic energy of translation possessed by each of the above molecules is:

$$
\begin{equation*}
\bar{u}_{p}=\frac{1}{2} \bar{m}_{i} \bar{v}_{p}^{2} \tag{TMA21}
\end{equation*}
$$

When the air is moving, that mean value is reduced by one-quarter of the amount in TMA19 averaged over $\boldsymbol{n}^{\downarrow}$. That gives us:

$$
\begin{equation*}
\Delta \bar{u}_{p}^{\downarrow}=-\frac{1}{2}\left(\operatorname{erf}\left(\frac{\bar{\omega}}{\sigma}\right)\right)\left(\bar{m}_{i} \bar{\omega}^{2}\right) \tag{TMA22}
\end{equation*}
$$

If our imaginary plane should suddenly become an impermeable sensing surface, molecules would no longer pass through the plane. Instead, they would impact upon the plane surface and transfer kinetic energy to that surface. The mean value of those molecular kinetic energy transfers would be sensed by that sensing surface as temperature. That temperature is shown by:

$$
\begin{equation*}
\bar{T}^{\downarrow}=\bar{T}-\frac{2 \Delta \bar{u}^{\downarrow}}{\mathbf{k}_{\mathrm{B}}}=\bar{T}-\frac{\left(\operatorname{erf}\left(\frac{\bar{\sigma}}{\sigma}\right)\right)\left(\bar{m}_{i} \bar{\Phi}^{2}\right)}{\mathbf{k}_{\mathrm{B}}} \tag{TMA23}
\end{equation*}
$$

## Temperature Variability at Other Wind Angles

In the above discussions, we have only looked at temperatures in moving air sensed by surfaces that are either parallel to or normal to the wind. What happens when the wind strikes a sensing surface at other angles?

Let us start our discussion by re-examining the angle of incidence. The angle of incidence alpha( $\boldsymbol{\alpha}$ ) is the angle that the wind vector makes with the sensing surface.

When the wind is normal to and toward the surface, $\boldsymbol{\alpha}$ is $+90^{\circ}$. When the wind is parallel to the surface, $\boldsymbol{\alpha}$ is $0^{\circ}$. When the wind is normal to and away from the surface, $\boldsymbol{\alpha}$ is $-90^{\circ}$. Intermediate angles will have intermediate values of $\boldsymbol{\alpha}$, ranging from $+90^{\circ}$ through $0^{\circ}$ to $-90^{\circ}$.

The derivation of the resulting temperatures is a simple exercise in trigonometry.

Temperatures at Positive Intermediate Angles of Incidence - When the angle of incidence is positive, the temperature sensed by the incident sensing surface is given by:

$$
\bar{T}^{\alpha}=(\sin \alpha) \bar{T}^{\rightarrow}+(1-\sin \alpha) \bar{T}^{\downarrow}
$$

TMA24

Temperatures at Negative Intermediate Angles of Incidence - When the angle of incidence is negative, the temperature is given by:

$$
\bar{T}^{\alpha}=(\sin \alpha) \bar{T}^{\leftarrow}+(1-\sin \alpha) \bar{T}^{\downarrow}
$$

TMA25

## Summary

Universal Gas Temperature Equation - Let us postulate a gas and a temperature-sensing surface (the sensor). The equation that defines the relationship between the gas and the sensor is:

$$
\begin{equation*}
\overline{\boldsymbol{T}}=\frac{\overline{\boldsymbol{u}}_{i}}{\mathbf{k}_{\mathrm{B}}} \tag{TMA01}
\end{equation*}
$$

Here, $\overline{\boldsymbol{T}}$ is the mean temperature of the gas (or mixture of non-reacting gases) in Kelvins as sensed by and measured by the sensor. The all-important $\overline{\boldsymbol{u}}_{\boldsymbol{i}}$ is the mean kinetic energy of translation normal to the sensor of that sub-population of molecules that actually impact on the sensor during that interval of time that produces that temperature. This kinetic energy is measured in joules at the instants of molecular impact. The constant of proportionality relating Kelvins to joules is $\mathbf{k}_{\mathbf{B}}$. This is Boltzmann's Constant in joules per Kelvin per molecule.

Being essentially a definition, this universal temperature equation is valid for ideal gases and for real gases, under conditions of equilibrium and under conditions of non-equilibrium, in the laboratory and in the free atmosphere, for still air and for moving air. It is even valid for temperatures sensed by moving objects in moving air.

As with all definitions, "the Devil is in the details". In this case, the Devil is $\overline{\boldsymbol{u}}_{\boldsymbol{i}}$. This is the mean molecular component kinetic energy of translation measured normal to the sensing surface at the instants of impact of those select molecules that actually made impact on the sensor during that interval of time necessary to create the temperature reading. No other molecules are involved. As we saw in The Two Populations, this interacting sub-population has parameters that are significantly different from those of the general population.

Kinetic Energy of Translation Equivalences - In still air, there are rigid proportions between the various parameters of kinetic energy of translation. These are examined in Kinetic Energies of Translation. From that source, Equations KET23 and KET28 give us the following relationships and equivalences:

$$
\begin{equation*}
\bar{u}_{i}=\bar{u}_{\sigma}=\mathbf{2} \bar{u}_{p}=\frac{2}{3} \overline{\boldsymbol{u}}=\frac{1}{2} \bar{u}_{i^{\prime}}=\mathbf{k}_{\mathrm{B}} \overline{\boldsymbol{T}} \tag{KET23}
\end{equation*}
$$

$$
\begin{equation*}
\bar{u}_{i}=\bar{m}_{i} \sigma^{2}=\bar{m}_{i} \overline{v_{p}^{2}}=\frac{1}{2} \bar{m}_{i} \overline{v_{i}^{2}}=\frac{1}{3} \bar{m}_{i} \overline{v^{2}}=\frac{1}{4} \bar{m}_{i} \overline{v_{i}^{2}} \tag{KET28}
\end{equation*}
$$

For definitions of the terms and their individual derivations, the reader is referred to Kinetic Energies of Translation.

Two other equivalences that the reader might find useful relate the means of the squares $\left(\overline{\boldsymbol{v}_{x}^{2}}\right)$ to the squares of the means $\left(\overline{\boldsymbol{v}}_{x}^{2}\right)$ of molecular component velocities:

$$
\begin{align*}
& \overline{v_{p}^{2}}=\frac{\pi}{2} \bar{v}_{p}^{2} \\
& \overline{v_{i}^{2}}=\frac{4}{\pi} \bar{v}_{i}^{2} \tag{TMA27}
\end{align*}
$$

TMA26

Temperatures Sensed by Windward Sensors - Thus armed with multiple versions of molecular kinetic energies and molecular velocities, let us summarize what happens to our still air molecules when the wind begins to blow. When this happens, both molecular velocities and molecular kinetic energies change in interesting ways.

We start by taking our imaginary sensing plane and orienting it so that the wind vector is normal to and toward the sensing surface. We then instruct our imaginary plane to freely pass all leeward molecules and to offer an impermeable thermometric surface to windward ones. The temperatures sensed by this windward surface will be:

$$
\begin{equation*}
\bar{T} \rightarrow=\bar{T}+\frac{\left(\bar{m}_{i} \bar{\omega}^{2}\right)}{\mathbf{k}_{\mathrm{B}}} \tag{TMA28}
\end{equation*}
$$

Temperatures Sensed by Leeward Sensors - When the wind is blowing, more molecules have a component of movement in the windward direction than have a component of movement in the leeward direction. Nevertheless, there are a substantial number in this latter category.

To measure their impacts on a sensing surface when the wind is normal to and away from that sensing surface, we instruct our imaginary plane to freely pass all
windward molecules and to offer an impermeable thermometric surface to leeward ones. The temperatures sensed by this surface will be:

$$
\begin{equation*}
\bar{T}^{\leftarrow}=\bar{T}-\frac{\left(\bar{m}_{i} \bar{\omega}^{2}\right)}{\mathbf{k}_{\mathrm{B}}} \tag{TMA29}
\end{equation*}
$$

Temperatures Sensed by Sensors Parallel to the Wind - We now rotate our imaginary sensing plane so that it is parallel to the wind vector. Half the molecules in the system will have a component of movement normal to and toward that surface and half will have a component of movement normal to and away from the surface. We now instruct our plane to offer an impermeable thermometric upper surface.

The molecules impacting on that surface from above will transfer that kinetic energy of translation normal to the surface to that surface. This transfer will be sensed as temperature. That temperature will be:

$$
\begin{equation*}
\bar{T}^{\downarrow}=\bar{T}-\frac{\left(\operatorname{erf}\left(\frac{\bar{\omega}}{\sigma}\right)\right)\left(\bar{m}_{i} \bar{\omega}^{2}\right)}{\mathbf{k}_{\mathrm{B}}} \tag{TMA30}
\end{equation*}
$$

At most normal wind speeds, this temperature decrement is far below detectable levels (see Molecular Parameter Values, below).

Temperatures Sensed by Sensors at Intermediate Wind Angles - The angle of incidence alpha $(\boldsymbol{\alpha})$ is the angle that the wind vector makes with the sensing surface. When the wind is normal to and toward the surface, $\boldsymbol{\alpha}$ is $+90^{\circ}$. When the wind is parallel to the surface, $\boldsymbol{\alpha}$ is $0^{\circ}$. When the wind is normal to and away from the surface, $\boldsymbol{\alpha}$ is $-90^{\circ}$. Intermediate angles will have intermediate values of $\boldsymbol{\alpha}$, ranging from $+90^{\circ}$ through $0^{\circ}$ to $-90^{\circ}$.

When the angle of incidence is positive, the temperature sensed by the incident sensing surface is given by:

$$
\begin{equation*}
\bar{T}^{\alpha}=(\sin \alpha) \bar{T}^{\rightarrow}+(1-\sin \alpha) \bar{T}^{\downarrow} \tag{TMA24}
\end{equation*}
$$

When the angle of incidence is negative, the temperature is given by:

$$
\begin{equation*}
\bar{T}^{\alpha}=(\sin \alpha) \bar{T}^{\leftarrow}+(1-\sin \alpha) \bar{T}^{\downarrow} \tag{TMA25}
\end{equation*}
$$

Note on Scale: The equations in this paper assume that the wind flows discussed are free of aberration. This is not an unreasonable assumption at the scale of molecular flows. Most of the molecular movements that we describe above originate no more than two or three molecular mean free paths from the sensing surface.

What we see as eddies and turbulence at the human scale is many orders of magnitude greater than the individual molecular movements on which our equations depend. Consequently, these macroscopic phenomena should be considered proper systems of study on their own merit, and not aberrations to the formulae.

## NUMERICAL PARAMETER VALUES

## Postulated:

| $\overline{\boldsymbol{T}}$ | $=298.15 \mathrm{~K}=25^{\circ} \mathrm{C}=77^{\circ} \mathrm{F}$ |
| :--- | :--- |
|  | $=$ Mean temperature of still air |
| $\overline{\boldsymbol{p}} \quad$ | $=1.0000 \times 10^{5}$ Pascals |
|  | Mean humid air pressure of still air |
| $\bar{\sim}$ |  |
|  | $=1.0000 \times 10^{3}$ Pascals |
|  | $=$ Mean water vapor pressure in still air |
| $\mathbf{k}_{\text {B }} \quad$ | $=1.3806488 \times 10^{-23}$ joules per Kelvin per molecule |
|  | $=$ Boltzmann's Constant |
| $\overline{\boldsymbol{\omega}} \quad$ | $=10.000$ meters per second |
|  | $=$ mean wind speed measured along the wind vector |

## Derived:

$\overline{\boldsymbol{n}} \quad=2.429305 \times 10^{25}$ humid air molecules per cubic meter $=$ mean molecular number density
$\overline{\boldsymbol{m}}_{i} \quad=4.77108 \times 10^{-26}$ kilograms
$=$ mean molecular impulse mass
$\sigma \quad=293.55$ meters per second
$=$ sigma. This is the root-mean-square $\left(v^{r m s}\right)$ of any and all axial molecular speed distributions in still air. It is also the standard deviation speed of those distributions, hence the use of the sigma term $(\boldsymbol{\sigma})$ in preference to the more awkward ( $\left.v^{r m s}\right)$ term.

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$\bar{v}_{p} \quad=234.22$ meters per second
$=$ In still air, this is the mean molecular velocity of the general population of molecules measured along our $\boldsymbol{p} / \boldsymbol{p}$ axis of movement.
$\bar{v}_{i} \quad=367.91$ meters per second
$=$ In still air, this is the mean molecular velocity of the interactive population of molecules measured along our $\boldsymbol{p} /-\boldsymbol{p}$ axis of movement.
$\overline{\boldsymbol{u}}_{\boldsymbol{p}} \quad=2.0582 \times 10^{-21}$ joules per molecule
$=$ In still air, this is the mean molecular kinetic energy of translation of the general population of molecules measured along our $p /-p$ axis of movement.
$\overline{\boldsymbol{u}}_{i} \quad=4.1164 \times 10^{-21}$ joules per molecule
$=$ In still air, this is the mean molecular kinetic energy of translation of the interactive population of molecules measured along our $\boldsymbol{p} / \boldsymbol{p}$ axis of movement.

## Wind-Related Parameters:

$\Delta \vec{n} \rightarrow \quad=3.3111 \times 10^{23}$ molecules per cubic meter
$=$ In moving air, this is the increment in the number of molecules of the general population having a component of movement in the wind direction.
$\Delta \overrightarrow{\boldsymbol{u}}_{p} \quad=-2.3855 \times 10^{-24}$ joules per molecule
$=$ In moving air, this is the windward increment in the mean molecular kinetic energy of translation of the general population of molecules, measured along our $\boldsymbol{p} /-\boldsymbol{p}$ axis of movement.
$\Delta \overrightarrow{\boldsymbol{u}}_{i} \quad=4.7711 \times 10^{-24}$ joules per molecule
$=$ In moving air, this is the windward increment in the mean molecular kinetic energy of translation of the interactive sub-population of molecules, measured along our $\boldsymbol{p} /-\boldsymbol{p}$ axis of movement.
$\Delta \overline{\boldsymbol{T}} \boldsymbol{r} \quad=0.34557$ Kelvins
$=$ In moving air, this is the mean windward increment in temperature sensed by a windward sensing surface normal to the wind vector.
$\Delta \overline{\boldsymbol{u}}_{-p}^{\leftarrow} \quad=-2.3855 \times 10^{-24}$ joules per molecule
$=$ In moving air, this is the leeward decrement in the mean molecular kinetic energy of translation of the general population of molecules, measured along our $\boldsymbol{p / - p}$ axis of movement.
$\Delta \bar{u}_{i}^{\leftarrow} \quad=-4.7711 \times 10^{-24}$ joules per molecule
$=$ In moving air, this is the leeward decrement in the mean molecular kinetic energy of translation of the interactive sub-population of molecules, measured along our $\boldsymbol{p} /-\boldsymbol{p}$ axis of movement.
$\Delta \overline{\boldsymbol{T}}^{\leftarrow} \quad=-0.34557$ Kelvins
$=$ In moving air, this is the mean leeward decrement in temperature sensed by a leeward sensing surface normal to the wind vector.
$\Delta \bar{u}_{p}^{\downarrow} \quad=6.5012 \times 10^{-26}$ joules per molecule
$=$ In moving air and in the general population of molecules having a component of movement normal to and toward the wind vector, this is the mean decrement in the associated component molecular kinetic energy of translation.
$\Delta \bar{u}_{i}^{\downarrow} \quad=1.3002 \times 10^{-25}$ joules per molecule =In moving air and in the interactive sub-population of molecules having a component of movement normal to and toward the wind vector, this is the mean decrement in the associated component molecular kinetic energy of translation.
$\Delta \overline{\boldsymbol{T}}^{\downarrow} \quad=-0.00942$ Kelvins
$=$ In moving air, this is the mean temperature decrement on a sensing surface parallel to the wind vector.
$\overrightarrow{\boldsymbol{n}} \quad=1.2478 \times 10^{25}$ molecules per cubic meter $=51.363 \%$ of $\overline{\boldsymbol{n}}$
$=$ This is the number of molecules per cubic meter having a component of movement in the windward direction.
 $=$ This is the number of molecules per cubic meter having a component of movement in the leeward direction.
$\boldsymbol{n}^{\downarrow} \quad=1.2147 \times 10^{25}$ molecules per cubic meter $=50.000 \%$ of $\overline{\boldsymbol{n}}$ $=$ This is the number of molecules per cubic meter having a component of movement normal to the wind vector.

## REFERENCES

INTERNAL REFERENCES: These are other papers in this collection that are either cited or linked during the course of the discussion or whose content is especially relevant to the current discussion.

The Three Principles - Establishes the three basic principles that guide the entire collection of papers. In a nutshell, these are: 1) Sensors only interact with a subpopulation of gas molecules. 2) This sub-population has parameters that differ from the general population. 3) Sensors only measure parameters that are normal to their surface.

One-Dimensional Space - Introduces the elegant and useful concept of onedimensional space. This sole spatial dimension is a measure of proximity. Molecules are either moving closer to our object of interest or are moving farther away. The speeds at which they are doing either of these two things can be measured. All molecular parameters (save those dealing with wind) may be expressed in terms based upon this one-dimensional space.

The Two Populations - Introduces the two populations of atmospheric molecules that acceptance of the second principle creates. These are the general population and the interacting population. This interacting population only comes into being when an object of interest is postulated, and exists only in relation to that object of interest.

Molecular Masses - This paper defines the molecular mass parameters ( $\overline{\boldsymbol{m}}, \overline{\boldsymbol{m}}_{\boldsymbol{i}}$ ) and shows how their values were calculated.

Molecular Speeds and Velocities - This absolutely essential paper defines the various velocity terms ( $\sigma, \overline{\boldsymbol{v}}_{p}, \overline{\boldsymbol{v}}_{i}$ ) used throughout this collection of papers and shows how they are derived and how they relate to one another mathematically.

THE KINETIC ATMOSPHERE
Wind-Induced Temperature Changes
Kinetic Energies of Translation - This paper is a fundamental paper in this collection. It defines the various molecular kinetic energies of translation ( $\overline{\boldsymbol{u}}, \overline{\boldsymbol{u}}_{\sigma}, \overline{\boldsymbol{u}}_{\boldsymbol{p}}, \overline{\boldsymbol{u}}_{\boldsymbol{i}}, \overline{\boldsymbol{u}}_{\boldsymbol{i}}$ ) and shows how each is derived. It goes on to show the mathematical relationships between the various energies and their relation to the thermal term ( $\left.\mathrm{k}_{\mathrm{B}} \overline{\boldsymbol{T}}\right)$.

Probability Density Curve - The Probability Density Equation, the mean value, standard deviation, history of the Probability Density Curve, the Normal Curve, velocity distribution along a single Cartesian axis, velocity distribution along the proximity axis, the Error Function Curve, The Complimentary Error Function Curve, proximity velocity curves, Table of Selected Values, evaluating the error functions.

Still Air Parameters - The value of common atmospheric parameters are calculated using a standard of 1,000 hectopascals of atmospheric pressure, a standard temperature of $25^{\circ} \mathrm{C}$, and a standard vapor pressure of 10 hectopascals. Values are then obtained for humid air, dry air, and water vapor. Parameters evaluated include: temperature, pressure, number density, mean molecular masses, mean molecular speeds and velocities, mean molecular flows, mean molecular momenta, specific density, mean molecular collisions with surfaces and with one another, mean free paths, mean molecular impulses, and mean kinetic energies of translation.

The Nature of Wind - This paper starts off with molecular flows and velocities in still air, then shows how these parameters change when the wind blows. The relationship of the normal curve to the distribution of flows and velocities is examined in detail. Molecular flows and velocities normal to the wind direction are derived. Tables and references.

EXTERNAL REFERENCES: These are papers by other authors that contain statements or data that are specifically incorporated into the current discussion.

Fundamental Physical Constants: 2006 CODATA - Peter J. Mohr, Barry N.
Taylor, and David B. Newell; National Institute of Science and Technology (NIST), CODATA Recommended Values of the Fundamental Physical
Constants: 2006; Gaithersburg, 2007.
http://physics.nist.gov/cuu/Constants/codata.pdf

THE KINETIC ATMOSPHERE
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