STILL AIR PARAMETERS

NORMALS: It is useful to have a single source for the value of common atmospheric parameters at some set of meteorological normals. This paper is intended to fill that need. I have chosen the value of 1,000 hectopascals (10^5 Pascals) as my normal for atmospheric pressure, the value of 10^3 Pascals as my normal for vapor pressure, and the value of 25°C (298.15K) as my normal for atmospheric temperature.

GOVERNING EQUATION: The governing equation used throughout this paper is:

$$\bar{p} = \bar{n}\mathrm{k_B}\bar{T} = \bar{n}\bar{m}_i\sigma^2 \qquad \text{SAP01}$$

Here, $\bar{p}$ is the mean atmospheric pressure in Pascals, $\bar{n}$ is the mean number density in number of molecules per cubic meter, $\mathrm{k_B}$ is Boltzmann's Constant in joules per molecule per Kelvin, $\bar{T}$ is the mean atmospheric temperature in Kelvins, $\bar{m}_i$ is the mean molecular impulse mass in kilograms, and σ (*sigma*) is the root-mean-square axial speed in meters per second. Sigma is also the **standard deviation** of the axial velocity distribution, hence the use of the conventional sigma symbol for standard deviation. Equation SAP01 is a simple combination of Equations GLF10 and GLF11 in *Gas Laws in the Free Atmosphere*.

NOTE1: A *tilde* beneath the parameter symbol ($\underset{\sim}{\bar{n}}$) means that the parameter refers solely to water vapor. A dot beneath the parameter symbol ($\underset{\cdot}{\bar{n}}$) means that the parameter refers solely to dry air. The absence of either ($\bar{n}$) means that the parameter refers to humid air.

NOTE2: For your convenience, a list of all parameters and their definitions is included at the end of this paper.

COMMON ASSUMPTIONS: Certain assumptions are common throughout this paper. The chief assumption is that the air is still. There is absolutely no net movement of air, no winds and no air currents. A corollary to this chief assumption is that there are no unbalanced changes of phase taking place. There are no clouds, no fog, no mist, and no precipitation. If a surface of water or ice is present, then that surface is in dynamic equilibrium with the overlying air. This means that any gross vaporization is exactly equaled by gross condensation; and that any melting of ice is exactly equaled by the freezing of water. If this were not so, then the volumetric changes consequent on the change of phase would induce airflow.

A second assumption is that the gases of the atmosphere behave essentially as ideal gases at the postulated values and the conditions of zero airflow and no changes of phase. This means that Equation SAP01 and all other equations of state under conditions of equilibrium will automatically balance. A moment's thought will show that this concept flows logically and naturally from the postulation of the temperature and pressure. By so doing, we have automatically defined the value of the remaining third state parameter, number density.

CONSTANTS: The following constant values have been used in calculations:

Unified atomic mass unit = u = 1.660 538 921 X 10^{-27} kilograms
Avogadro's number = N_A = 6.022 141 29 X 10^{23} molecules per mole
Boltzmann's constant = k_B = 1.380 648 8 X 10^{-23} joules per molecule per Kelvin

All values are from the 2010 CODATA (Mohr, *et al*, 2012).

TEMPERATURE

General: Temperature measures the mean kinetic energy of translation normal to and relative to the sensing surface of that selected sub-population of molecules that impacted on that surface during the period of time that affected the measurement.

Parameter Value:

$\bar{T}$ = 298.15 Kelvins.

Data Source: Postulated.

Governing Equation: $\quad \bar{T} = \dfrac{\bar{p}}{\bar{n}k_B} = \dfrac{\bar{m}_i \sigma^2}{k_B}$

Universal Temperature Equation: $\bar{T} = \dfrac{\bar{u}_i}{k_B}$ $\hspace{2cm}$ SAP02

Here, $\bar{u}_i$ is the mean impulse kinetic energy of translation (measured normal to the sensing surface in joules) of that selected sub-population of gas molecules that impacted upon that sensing surface during the period of time that affected the measurement. This equation is valid for ideal gases and for real gases, under conditions of equilibrium and under conditions of non-equilibrium. It is even valid for air temperatures sensed by moving objects in moving air.

Discussion: *The Nature of Atmospheric Temperatures.*

PRESSURE

General: Pressure measures the simple product of the mean number of molecular impacts per square meter per second and the mean impulse transferred (in newtons) of those impacts.

Parameter Values:

$\bar{p} = 10^5$ Pascals

$\bar{p}_{\sim} = 10^3$ Pascals

$\bar{p}_{\cdot} = 9.9 \times 10^4$ Pascals

Here, $\bar{p}_{\sim}$ is the mean vapor pressure and $\bar{p}_{\cdot}$ is the mean dry air pressure.

Data Sources: Postulated, postulated, and calculated from Dalton's Equation.

Governing Equation: $\bar{p} = \bar{n}k_B\bar{T} = \bar{n}\bar{m}_i\sigma^2$

Universal Pressure Equation: $\bar{p} = \bar{f}_i\bar{\varphi}_i$ 　　　　　　　　　　　SAP03

Here, $\bar{f}_i$ is the universal molecular flux in the mean number of molecular impacts per square meter per second, and $\bar{\varphi}_i$ is the universal mean molecular impulse per molecular impact in newtons. This equation is valid for ideal gases and for real gases, under conditions of equilibrium and under conditions of non-equilibrium. It is even valid for air pressures sensed by moving objects in moving air.

Daltons Equation: $\bar{p} = \bar{p}_{\sim} + \bar{p}_{\cdot}$ 　　　　　　　　　　　SAP04

Discussion: *The Nature of Atmospheric Pressures.*

NUMBER DENSITY

General: Number density is based on Avogadro's Law that states that for any given combination of temperature and pressure, the number of molecules of any gas or any combination of non-reacting gases is the same.

Parameter Values:

$\overline{n}$ = 2.429 305 X 10^{25} humid air molecules per cubic meter.

$\overline{\underset{\sim}{n}}$ = 2.429 305 X 10^{23} water vapor molecules per cubic meter.

$\overline{\overset{\cdot}{n}}$ = 2.405 012 X 10^{25} dry air molecules per cubic meter

Data Sources: Algebraic solutions of the Governing Equation and Avogadro's Identity.

Governing Equation: $\overline{n} = \dfrac{\overline{p}}{k_B \overline{T}}$

Universal Number Density Equation: $\overline{n} = \dfrac{\overline{\rho}}{\overline{m}}$
SAP05

Here, $\overline{\rho}$ is the mean mass density in kilograms per cubic meter, and $\overline{m}$ is the mean molecular mass of the general population of humid air molecules. This equation is valid for ideal gases and for real gases, under conditions of equilibrium and under conditions of non-equilibrium. It is even valid for mass densities in parcels of moving air.

Avogadro's Identity: $\overline{n} = \overline{\underset{\sim}{n}} + \overline{\overset{\cdot}{n}}$
SAP06

Discussion: *Molecular Numbers* and *Atmospheric Densities*.

MOLECULAR MASSES

General: Mean molecular masses are calculated from estimates of the atmospheric abundance of the various isotopes of the most common atmospheric gases and estimates of the various isotopic masses. The following values were calculated from the forty-four (44) dry air isotopes that are present in parts per billion or greater concentrations and the nine (9) water vapor isotopes.

Parameter Values:

$\overline{\overline{m}}$ = 4.791 668 X 10^{-26} kilograms

$\overline{\underset{\bullet}{m}}$ = 4.809 851 X 10^{-26} kilograms

$\overline{\underset{\sim}{m}}$ = 2.991 505 X 10^{-26} kilograms

$\overline{m}_i$ = 4.777 108 X 10^{-26} kilograms

$\overline{\underset{\bullet}{m}}_i$ = 4.795 145 X 10^{-26} kilograms

$\overline{\underset{\sim}{m}}_i$ = 2.991 450 X 10^{-26} kilograms

Here, $\overline{\underset{\bullet}{m}}$ is the mean molecular mass of the dry air components, $\overline{\underset{\sim}{m}}$ is the mean molecular mass of the water vapor component, $\overline{\underset{\bullet}{m}}_i$ is the mean interactive molecular mass of the dry air components, and $\overline{\underset{\sim}{m}}_i$ is the mean interactive molecular mass of the water vapor component.

Data Sources: *Molecular Masses* and computations from

$$\overline{\overline{m}} = \frac{(\overline{\underset{\bullet}{m}}\,\overline{\underset{\bullet}{n}}) + (\overline{\underset{\sim}{m}}\,\overline{\underset{\sim}{n}})}{\overline{\overline{n}}}$$
<div style="text-align:right">SAP07</div>

Discussion: *Molecular Masses.*

MOLECULAR SPEEDS AND VELOCITIES

General: At any given temperature, each and every value of molecular mass (m) will have its own distribution of probable values for the resulting molecular speeds and velocities (see the *Governing Equation*, below). Each of these individual distributions will have its own means, modes, medians, and standard deviations. Nevertheless, the numerical relationships between these consequent parameters will always be the same. For instance, the **relationship** between the standard deviation of axial velocities (σ) and the mean molecular velocity along the true paths ($\overline{v}$) will always have the same numerical value ($\sigma = \frac{1}{2}\sqrt{\frac{\pi}{2}}\overline{v}$) regardless of the fact the parameters themselves will have different values. This is true for all other mean molecular speed and velocity parameters as well. This enables us to use common equations to describe the molecular speeds and velocities of the humid atmosphere despite the fact that this atmosphere is composed of several hundred different molecular speed and velocity distributions.

Parameter Values:

Parameters of the general population of humid air molecules:

σ = 293.55 m sec^{-1} This is the root-mean-square speed of any and all axial molecular speed distributions. It is also the standard deviation of those distributions. All molecular mean speeds bear a fixed relationship to **sigma**.

$\overline{v}_p = \sqrt{\frac{2}{\pi}}\sigma$ = 234.22 m sec^{-1} This is the mean molecular velocity measured normal to and towards any object of interest.

$\overline{v} = 2\sqrt{\frac{2}{\pi}}\sigma$ = 468.43 m sec^{-1} This is the mean molecular velocity along the true paths.

Parameters of the interactive sub-population of humid air molecules:

$\overline{v}_i = \sqrt{\frac{\pi}{2}}\sigma$ = 367.91 m sec^{-1} This is the mean molecular velocity measured normal to and towards any object of interest at the moment of interaction.

$\overline{v}_{i'} = \frac{3}{2}\sqrt{\frac{\pi}{2}}\sigma$ = 551.86 m sec^{-1} This is the mean molecular velocity along the true paths at the moment of interaction.

Parameters of the general population of water vapor molecules:

$\underset{\sim}{\sigma}$ = 370.95 m sec^{-1} This is the root-mean-square speed of any and all axial molecular speed distributions. It is also the standard deviation of those distributions. All molecular mean speeds bear a fixed relationship to **sigma**.

$\underset{\sim}{\overline{v}}_p = \sqrt{\frac{2}{\pi}}\underset{\sim}{\sigma}$ = 295.98 m sec^{-1} This is the mean molecular velocity measured normal to and towards any object of interest.

$\underset{\sim}{\overline{v}} = 2\sqrt{\frac{2}{\pi}}\underset{\sim}{\sigma}$ = 591.95 m sec^{-1} This is the mean molecular velocity along the true paths.

Parameters of the interactive sub-population of water vapor molecules:

$\underset{\sim}{\overline{v}}_i = \sqrt{\frac{\pi}{2}}\underset{\sim}{\sigma}$ = 464.92 m sec^{-1} This is the mean molecular velocity measured normal to and towards any object of interest at the moment of interaction.

$\underset{\sim}{\overline{v}}_{i'} = \frac{3}{2}\sqrt{\frac{\pi}{2}}\underset{\sim}{\sigma}$ = 697.38 m sec^{-1} This is the mean molecular velocity along the true paths at the moment of interaction.

Parameters of the general population of dry air molecules:

$\overset{\bullet}{\sigma}$ = 292.99 m sec^{-1} This is the root-mean-square speed of any and all axial molecular speed distributions. It is also the standard deviation of those distributions. All molecular mean speeds bear a fixed relationship to **sigma**.

$\overline{v}_p = \sqrt{\frac{2}{\pi}}\overset{\bullet}{\sigma}$ = 233.78 m sec^{-1} This is the mean molecular velocity measured normal to and towards any object of interest.

$\overline{v} = 2\sqrt{\frac{2}{\pi}}\overset{\bullet}{\sigma}$ = 467.55 m sec^{-1} This is the mean molecular velocity along the true paths.

Parameters of the interactive sub-population of dry air molecules:

$\overline{v}_i = \sqrt{\frac{\pi}{2}}\overset{\bullet}{\sigma}$ = 367.21 m sec^{-1} This is the mean molecular velocity measured normal to and towards any object of interest at the moment of interaction.

$\overline{v}_{i'} = \frac{3}{2}\sqrt{\frac{\pi}{2}}\overset{\bullet}{\sigma}$ = 550.82 m sec^{-1} This is the mean molecular velocity along the true paths at the moment of interaction.

Data Sources: Calculations from governing equation, the equation of equivalence, and the calculated mean molecular masses.

Governing Equation: $\sigma = \left(\dfrac{k_B \overline{T}}{\overline{m}_i} \right)^{1/2}$

Equation of Equivalence: $\sigma = \sqrt{\frac{\pi}{2}}\overline{v}_p = \sqrt{\frac{2}{\pi}}\overline{v}_i = \frac{1}{2}\sqrt{\frac{\pi}{2}}\overline{v} = \frac{2}{3}\sqrt{\frac{2}{\pi}}\overline{v}_{i'}$ SAP08

Discussion: *Molecular Speeds and Velocities*

MOLECULAR FLOWS

General: Molecular flows measure the number of molecules that pass through a plane (real or imaginary) or impact upon a surface (real or imaginary) per square meter per second.

Parameter Values:

$$\overline{f}_i^e = \sqrt{\tfrac{1}{2\pi}}\,\overline{n}\sigma = 1.78\ 864 \text{ X } 10^{28} \text{ interactions per square meter per second}$$

$$\underset{\sim}{\overline{f}}_i^e = \sqrt{\tfrac{1}{2\pi}}\,\underset{\sim}{\overline{n}}\,\underset{\sim}{\sigma} = 2.25\ 886 \text{ X } 10^{28} \text{ interactions per square meter per second}$$

$$\underset{.}{\overline{f}}_i^e = \sqrt{\tfrac{1}{2\pi}}\,\underset{.}{\overline{n}}\,\underset{.}{\sigma} = 1.76\ 630 \text{ X } 10^{28} \text{ interactions per square meter per second}$$

Here, $\overline{f}_i^e$ is the mean number of passages or impacts (under conditions of equilibrium) per square meter per second, $\overline{n}$ is the mean number of molecules in the population of molecules, and σ is the root-mean-square axial speed of each and every axial speed distribution.

Data Sources: Derived from the governing equation using calculated values of mean molecular number density and the root-mean-square axial speed.

Governing Equation: $\overline{f}_i^e = \sqrt{\tfrac{1}{2\pi}}\,\overline{n}\sigma$

Universal Flux Equation: $\overline{f}_i = \overline{n}_p \overline{v}_p = \dfrac{\overline{p}}{\overline{\varphi}_i}$ SAP09

Here, $\overline{f}_i$ is the mean number of passages or impacts per square meter per second, $\overline{n}_p$ is the mean number of molecules in the sub-population of those molecules that have a relative component of motion toward the object of interest, and $\overline{v}_p$ is the mean molecular speed in meters per second of that component relative to the object of interest, $\overline{p}$ is the mean gaseous pressure

upon the object of interest in Pascals, and $\overline{\varphi}_i$ is the mean impulse per impact in newtons. This equation is valid for ideal gases and for real gases, under conditions of equilibrium and under conditions of non-equilibrium. It is even valid for the flux upon moving objects in moving air.

Equilibrium Flux Equation: $\overline{f}_i^{\,e} = \tfrac{1}{2}\overline{n}\overline{v}_p$ 　　　　　　　　　SAP10

Discussion: *Molecular Flows*.

MOLECULAR MOMENTA

General: Molecular momentum measures the simple product of an individual molecule's mass and its velocity. As we have seen, however, molecular populations have both an intrinsic mass ($\overline{m}$) and an interactive mass ($\overline{m}_i$) and each population has two different mean velocities ($\overline{v}_p, \overline{v}_i, \overline{v}, \overline{v}_{i'}$). Consequently, there can be four different mean molecular momenta for a population of molecules.

Fortunately, only one of these four mean momenta plays any role in statistical mechanics and that is $\overline{m}_i\overline{v}_i$. That is the only momentum that can be transferred from one molecule to another or from a molecule to a surface. This is because momentum transfer requires that the molecules interact ($\overline{m}_i$) and only the velocity normal to and toward the object of interest at the moment of interaction ($\overline{v}_i$) can be transferred. Note that at the instant of impact, the momentum ceases to be momentum and becomes **impulse**.

Momentum that fails to become impulse plays no role in the kinetic atmosphere.

Parameter Values:

$\overline{\rho} = 1.75\ 642 \times 10^{-23}$ kilogram-meters per second

$\overrightarrow{\overline{\rho}} = 1.39\ 078 \times 10^{-23}$ kilogram-meters per second

$\underset{\rightrightarrows}{\overline{\rho}} = 1.76\ 084 \times 10^{-23}$ kilogram-meters per second

Here, $\underset{\rightarrow}{\overline{\rho}}$ is the mean molecular momentum at the instant of interaction.

Data Sources: Solution of the Governing Equation using the calculated values of the interactive masses and constants and the postulated values of the other variables.

Governing Equation: $\overrightarrow{\overline{\rho}} = \left(\dfrac{\pi \overline{m}_i k_B \overline{T}}{2} \right)^{1/2}$

Universal Equation: $\overrightarrow{\overline{\rho}} = \overline{m}_i \overline{v}_i$ 　　　　　　　　　　　　 SAP11

This equation is valid for ideal gases and for real gases, under conditions of equilibrium and under conditions of non-equilibrium. It is even valid for mean molecular momenta in moving air.

Other Equation: $\overline{v}_i = \sqrt{\tfrac{\pi}{2}} \sigma$ 　　　　　　　　　　　　 SAP12

Discussion: *Molecular Momenta*

SPECIFIC DENSITY

General: The mean specific density of a parcel of atmospheric air is the simple product of the mean number density and the mean molecular mass. Although widely used in fluid dynamics, it is rarely used at all in either kinetic gas theory or statistical mechanics. In these two disciplines, it does not appear to be a fertile concept. Nevertheless, here are the parameter values for whatever they are worth.

Parameter Values:

$\overline{\rho}$ = 1.15 904 kilograms per cubic meter

$\underset{\sim}{\overline{\rho}}$ = 7.26 714 X 10^{-3} kilograms per cubic meter

$\underset{\cdot}{\overline{\rho}}$ = 1.15 324 kilograms per cubic meter

Data Sources: Solution of the universal equation using calculated values of mean number density and mean molecular mass.

Universal Equation: $\overline{\rho} = \overline{n}\,\overline{m}$

Here, $\overline{\rho}$ is the specific density in kilograms per cubic meter, $\overline{n}$ is the mean number density in number of molecules per cubic meter, and $\overline{m}$ is the mean molecular mass in kilograms. This equation is valid for ideal gases and for real gases, under conditions of equilibrium and under conditions of non-equilibrium. It is even valid for the mean specific density of parcels of moving air.

Discussion: *Atmospheric Densities*

INTERMOLECULAR COLLISIONS

General: It is a basic premise of both kinetic gas theory and statistical mechanics that molecules collide with one another and with any exposed surface. The frequency with which molecules collide with an exposed surface has already been dealt with in the section on **Molecular Flows**, above. There we saw that the Governing Equation took the form:

$$\overline{f}_i^e = \sqrt{\tfrac{1}{2\pi}}\,\overline{n}\sigma$$

Here, $\overline{f}_i^e$ is the number of molecular impacts per square meter per second under conditions of equilibrium. To calculate the number of intermolecular collisions, we need to come up with a molecular surface area.

CONTACT COLLISIONS: In an ideal gas composed of rigid, spherical, elastic molecules, whenever the distance between the center of mass of the first molecule reaches a distance of $\left(r_1 + r_2\right)$ from the center of mass of the second molecule (where r is the respective molecular radius) intermolecular contact may be presumed and a contact collision takes place. This is analogous to a point mass approaching a sphere whose radius is $\left(r_1 + r_2\right)$. The surface area of such a sphere is:

$$\otimes = 4\pi\left(r_1 + r_2\right)^2 \qquad\qquad \text{SAP14}$$

Therefore, the frequency of intermolecular contact collision will be:

$$\overline{f}_\otimes^e = \sqrt{\tfrac{1}{2\pi}}\,\overline{n}\sigma\,\overline{\otimes} \qquad\qquad \text{SAP15}$$

Here, $\overline{f}_\otimes^e$ is the number of intermolecular contact collisions per molecule per second and $\overline{\otimes}$ is the mean molecular surface area.

Real air molecules are not spherical, but we may assume that the various molecular rotations, vibrations, and librations "clear out" a volume that is roughly spherical. Unfortunately, this still leaves us with the question of what is the mean molecular "size" for the many different molecules of the Earth's atmosphere. This is a field where little is known with any kind of precision.

Consequently, we will approximate the following values for molecular radii:

$\overline{\underset{\sim}{r}} \approx 1.55$ X 10^{-10} meters for the mean radius of a water vapor molecule

$\overline{\underset{\cdot}{r}} \approx 1.90$ X 10^{-10} meters for the mean radius of a dry air molecule

$\overline{r} \approx 1.89$ X 10^{-10} meters for the mean radius of a humid air molecule

NON-CONTACT COLLISIONS: Properly speaking, we should define an intermolecular collision as any intermolecular interaction that results in an exchange of momentum; i. e., either or both molecules undergoes a change in direction or speed of movement. Such non-contact "collisions" are frequently caused by the electrostatic forces of ions and polar molecules and the unsaturated hydrogen bond of water vapor molecules.

The parameter values below do not take such "collisions" into account. Hence, we must recognize that these values significantly underestimate the reality of intermolecular collisions.

Parameter Values:

$\overline{f}_{\otimes}^{e} \approx 5.11$ X 10^{9} intermolecular collisions per humid air molecule per second

Data Source: Solution of the Governing Equation using the estimated value for mean molecular surface area.

Governing Equation: $\overline{f}_{\otimes}^{e} = \sqrt{\frac{1}{2\pi}}\,\overline{n}\,\sigma\,\overline{\otimes}$

Here, $\overline{f}_{\otimes}^{e}$ is the mean number of intermolecular collisions per cubic meter per second, $\overline{n}$ is the mean number density in number per cubic meter, σ is the root-mean-square molecular axial velocity in meters per second, and $\overline{\otimes}$ is the mean molecular surface area in square meters.

Universal Equation:

$$\overline{f}_{\overline{\otimes}} = \overline{n}_{p}\overline{v}_{p}\,\overline{\otimes} \qquad\qquad \text{SAP16}$$

Here, $\overline{n}_p$ is the mean number of molecules per cubic meter that have a component of movement toward the object of interest and $\overline{v}_p$ is the mean axial velocity of that component.

Other Useful Equations:

$$\overline{f}_\Sigma^e = \overline{f}_\otimes^e \frac{\overline{n}}{2} = 6.21 \times 10^{34} \text{ total intermolecular collisions m}^{-3} \text{ sec}^{-1} \qquad \text{SAP17}$$

$$\overline{f}_{\overline{\otimes}} = \overline{\underset{\sim}{n}}_p \underset{\sim}{\overline{v}}_p \; \overline{\underset{\sim}{\otimes}} \qquad \text{SAP18}$$

This last equation is useful in condensation studies.

Discussion: *Molecular Collisions*

MEAN FREE PATHS

General: The mean free path is the average distance that a molecule travels between collisions. It is usually obtained by dividing the molecules' mean velocity by the mean frequency of intermolecular collisions. Since all free paths take place between collisions, the only molecular velocities that we have to concern ourselves with are the two mean impulse velocities, $\overline{v}_i$ and $\overline{v}_{i'}$.

MEAN AXIAL FREE PATH: This is the mean distance, measured normal to the molecules involved, that a molecule travels between intermolecular collisions.

$$\overline{\lambda}_i = \frac{\overline{v}_i}{\overline{f}_\otimes} \qquad \text{SAP19}$$

Here, $\overline{\lambda}_i$ is the mean free path in meters between collisions measured along the proximity axis, $\overline{v}_i$ is the mean axial arm speed in meters per second between the first and the second of any two subsequent collisions measured along the proximity axis, and $\overline{f}_\otimes$ is the frequency of intermolecular collisions in number per molecule per second.

MEAN TRUE FREE PATH: This is the mean distance that a molecule travels along its true path between intermolecular collisions.

$$\overline{\lambda}_{i'} = \frac{\overline{v}_{i'}}{\overline{f}_\otimes} \qquad \text{SAP20}$$

Here, $\overline{\lambda}_{i'}$ is the distance traveled in meters between collisions measured along the molecule's true path, and $\overline{v}_{i'}$ is the mean speed in meters per second between the first and the second of any two subsequent collisions measured along the molecule's true path.

Since $\overline{v}_{i'}$ is exactly 3/2 greater than $\overline{v}_i$, $\overline{\lambda}_{i'}$ is also 3/2 greater than $\overline{\lambda}_i$.

Governing Equations: $\overline{\lambda}_i = \dfrac{\pi}{\overline{n}\,\overline{\otimes}}$ and $\overline{\lambda}_{i'} = \dfrac{3\pi}{2\overline{n}\,\overline{\otimes}}$

Parameter Values:

$\bar{\lambda}_i$ = 2.86 X 10^{-7} meters

$\bar{\lambda}_{i'}$ = 4.29 X 10^{-7} meters

Data Source: Solution of the Governing Equations using calculated values of mean number density and mean molecular surface area.

Discussion: *Mean Free Paths*

MOLECULAR IMPULSES:

General: When a real air molecule impacts upon a real surface, that impact is not instantaneous. It occupies a real segment of time. During that time, a force is transmitted to that surface by that molecule's impact. That force is impulse, and it is measured in newtons.

$$\varphi_i = \Delta\rho_{\rightarrow} = \frac{d\vec{\rho}}{dt} = \int_{initial\ contact}^{final\ contact} m_i v_i \qquad\qquad \text{SAP21}$$

Here, φ_i is the impulse in newtons, $\underset{\rightarrow}{\rho}$ is the momentum in kilogram-meters per second, m_i is the impulse molecular mass, and v_i is the impulse velocity normal to and toward the surface at the instant of impact. The impulse is equal to the change in momentum over the time of impact. Its dimensions are MLT^{-2}.

IDEAL GAS IMPULSES: In an ideal gas where all collisions are perfectly elastic the molecule will rebound with a momentum equal in magnitude but opposite in direction to its initial momentum. Thus, the impulse will be:

$$\overline{\varphi}_i = 2\overline{m}_i \overline{v}_i \qquad\qquad \text{SAP22}$$

REAL GAS IMPULSES: In the real gases of the atmosphere, many collisions are not perfectly elastic. Molecules may adhere to or be absorbed by the surface. They may be ionized by the collision and gain or lose one or more electrons, thereby changing both their mass and their momentum. They may gain or lose velocity by collision with a hotter or colder surface. This is just a sampling of real-world interactions between a molecule and the surface. All of them render equation SAP22 invalid.

Although these deviations are common, they are not usually so common as to cause equation SAP22 to be invalid at the normal level of scientific accuracy and precision. Don't expect much more than the proper order of magnitude and three significant figures and you won't be disappointed—unless changes of phase

are taking place. If that is the case, then all bets are off. Equation SAP22 will not be valid.

Governing Equation: $\overline{\varphi}_i = \sqrt{2\pi \overline{m}_i \sigma}$

Parameter Value: 2.20 X 10^{-23} newtons per impact

Data Source: Solution of the Governing Equation using calculated values of mean molecular impulse mass and *sigma*.

Universal Equation: $\overline{\varphi}_i = \dfrac{\overline{p}}{\overline{f}_i}$ SAP23

Here, $\overline{\varphi}_i$ is the mean molecular impulse in newtons, $\overline{p}$ is the mean gaseous

pressure upon the object of interest in Pascals, and $\overline{f}_i$ is the mean frequency of impact in number per square meter per second. This equation is valid for ideal gases and for real gases, under conditions of equilibrium and under conditions of non-equilibrium. It is even valid for the mean molecular impulse felt by moving objects in moving air.

Discussion: *Molecular Impulses*

MOLECULAR KINETIC ENERGIES OF TRANSLATION

General: Classical mechanics defines the total kinetic energy of translation of a population of ideal gas molecules as:

$$\bar{u} = \tfrac{1}{2} m \overline{v^2} \qquad\qquad \text{SAP24}$$

Here, $\bar{u}$ is the mean kinetic energy of translation over the various true paths, m is the singular molecular mass, and $\overline{v^2}$ is the mean of the squares of the individual molecule's velocities over those true paths. Component kinetic energies of translation along some particular axis will use the same formula, with the proper axial subscripts for the energy parameter and the velocity parameter. For gases with multiple molecular masses, m must be replaced by mean impulse mass, $\bar{m}_i$:

$$\bar{u} = \tfrac{1}{2} m_i \overline{v^2} \qquad\qquad \text{SAP25}$$

As we saw in the section above on *Molecular Speeds and Velocities*, a population of molecules will have five different mean speeds whose proportions to one another are shown by the Equation of Equivalence:

Equation of Speed Equivalence:

$$\sigma = \sqrt{\frac{k_B \bar{T}}{\bar{m}_i}} = \sqrt{\frac{\pi}{2}}\,\bar{v}_p = \sqrt{\frac{2}{\pi}}\,\bar{v}_i = \tfrac{1}{2}\sqrt{\frac{\pi}{2}}\,\bar{v} = \tfrac{2}{3}\sqrt{\frac{2}{\pi}}\,\bar{v}_{i'} \qquad\qquad \text{SAP08}$$

Here, σ is the root-mean-square (v_{rms}) axial speed in meters per second, k_B is Boltzmann's Constant in joules per degree per molecule, $\bar{T}$ is the mean gas temperature in Kelvins, $\bar{v}_p$ is the mean axial speed of the general population of gas molecules, $\bar{v}_i$ is the mean axial speed of the interactive population of gas molecules, $\bar{v}$ is the mean speed along their true paths of the general population of gas molecules, and $\bar{v}_{i'}$ is the mean speed along their true paths of the interactive population of gas molecules.

Each of the five above speeds will have its own kinetic energy of translation:

Equations of Kinetic Energy of Translation Equivalence:

$$\mathbf{k_B}\overline{T} = \overline{m}_i\sigma^2 = \overline{u}_i = 2\overline{u}_p = \tfrac{2}{3}\overline{u} = \tfrac{1}{2}\overline{u}_{i'} \qquad\qquad \text{SAP26}$$

Here, $\mathbf{k_B}$ is Boltzmann's Constant in joules per molecule per Kelvin, $\overline{T}$ is the mean gas temperature in Kelvins, σ^2 is the square of the root-mean-square axial speed in meters per second, $\overline{u}_i$ is the mean axial kinetic energy of translation normal to and toward any object of interest, $\overline{u}_p$ is the mean axial kinetic energy of translation in the general population of gas molecules, $\overline{u}$ is the mean true path kinetic energy of translation of the general population of gas molecules, and $\overline{u}_{i'}$ is the mean true path kinetic energy of translation of the interactive population of gas molecules.

$$\tfrac{1}{2}\mathbf{k_B}\overline{T} = \tfrac{1}{2}\overline{m}_i\sigma^2 = \tfrac{1}{2}\overline{m}_i\overline{v_p^2} = \tfrac{1}{4}\overline{m}_i\overline{v_i^2} = \tfrac{1}{6}\overline{m}_i\overline{v^2} = \tfrac{1}{8}\overline{m}_i\overline{v_{i'}^2} \qquad\qquad \text{SAP27}$$

Here, $\mathbf{k_B}$ is Boltzmann's Constant in joules per molecule per Kelvin, $\overline{T}$ is the mean gas temperature in Kelvins, σ^2 is the square of the root-mean-square axial speed in meters per second, $\overline{v_p^2}$ is the mean square of the axial speeds of the general population of gas molecules, $\overline{v_i^2}$ is the mean square of the axial speeds of the interactive population of gas molecules, $\overline{v^2}$ is the mean square of the true path speeds of the general population of gas molecules, and $\overline{v_{i'}^2}$ is the mean square of the true path speeds of the interactive population of gas molecules.

Governing Equation: $\overline{u}_i = \mathbf{k_B}\overline{T}$

Parameter Values:

$\overline{u}_p = \tfrac{1}{2}\mathbf{k_B}\overline{T}$ = 2.06 X 10^{-21} joules per molecule

$\overline{u}_i = \mathbf{k_B}\overline{T}$ = 4.12 X 10^{-21} joules per molecule

$\bar{u} = \frac{3}{2}\mathbf{k_B}\bar{T}$ = 6.17 X 10^{-21} joules per molecule

$\bar{u}_{i'} = 2\mathbf{k_B}\bar{T}$ = 8.23 X 10^{-21} joules per molecule

Data Sources: Solution of equation SAP26 using postulated values of Boltzmann's Constant and temperature.

Discussion: *Kinetic Energies of Translation*

DEFINITIONS OF PARAMETERS USED IN THIS PAPER

$\overline{f}_i$ **=** the mean molecular flow rate in number of interactions per square meter per second for humid air

$\overline{f}_{\underset{\sim}{i}}$ **=** the mean molecular flow rate in number of interactions per square meter per second for water vapor

$\overline{f}_{\underset{\cdot}{i}}$ **=** the mean molecular flow rate in number of interactions per square meter per second for water vapor

$\mathbf{k_B}$ = Boltzmann's Constant in joules per molecule per Kelvin.

$\overline{m}$ = mean molecular mass in kilograms.

$\overline{m}_{\cdot}$ = mean molecular mass in kilograms of the dry air components.

$\overline{m}_{\sim}$ = mean molecular mass in kilograms of the water vapor component.

$\overline{m}_i$ = mean interactive molecular mass in kilograms.

$\overline{m}_{\underset{\cdot}{i}}$ = mean interactive molecular mass in kilograms of the dry air components.

$\overline{m}_{\underset{\sim}{i}}$ = mean interactive molecular mass in kilograms of the water vapor component.

$\overline{n}$ = mean molecular number density in number per cubic meter.

$\overline{n}_{\sim}$ = mean water vapor molecular number density in number per cubic meter.

$\overline{n}_{\cdot}$ = mean dry air molecular number density in number per cubic meter.

$\overline{p}$ = mean atmospheric pressure in Pascals.

$\overline{p}_{\sim}$ = mean atmospheric vapor pressure in Pascals.

$\overline{p}_{\cdot}$ = mean atmospheric dry air pressure in Pascals.

$\overline{r}$ = mean molecular radius for humid air

$\overline{r}_{\sim}$ = mean molecular radius for water vapor

$\overline{r}_{\cdot}$ = mean molecular radius for dry air

$\overline{T}$ = mean atmospheric temperature in Kelvins.

$\overline{u}_i$ = mean molecular interactive kinetic energy of translation (in joules and measured normal to the sensing surface) of that sub-population of

molecules that interacted with the sensing surface during the period of measurement.

$\overline{v}_p$ = mean molecular velocity measured normal to and towards any object of interest.

$\overline{v}$ = mean molecular velocity along the true paths.

$\overline{v}_i$ = mean molecular velocity measured normal to and towards any object of interest at the instant of interaction.

$\overline{v}_{i'}$ = mean molecular velocity along the true paths at the instant of interaction.

$\overline{v}_{\sim p}$ = mean water vapor molecular velocity measured normal to and towards any object of interest.

$\overline{v}_{\sim}$ = mean water vapor molecular velocity along the true paths.

$\overline{v}_{\sim i}$ = mean water vapor molecular velocity measured normal to and towards any object of interest at the instant of interaction.

$\overline{v}_{\sim i'}$ = mean molecular velocity along the true paths at the instant of interaction.

$\overline{v}_{\cdot p}$ = mean dry air molecular velocity measured normal to and towards any object of interest.

$\overline{v}_{\cdot}$ = mean dry air molecular velocity along the true paths.

$\overline{v}_{\cdot i}$ = mean molecular velocity measured normal to and towards any object of interest at the moment of interaction.

$\overline{v}_{\cdot i'}$ = mean molecular velocity along the true paths at the instant of interaction

$\overline{\otimes}$ = mean molecular effective surface area in square meters for humid air

$\overline{\otimes}_{\sim}$ = mean molecular effective surface area in square meters for water vapor

$\overline{\otimes}_{\cdot}$ = mean molecular effective surface area in square meters for dry air

σ = **sigma**, the root-mean-square axial molecular speed in meters per second. This value is also the standard deviation of the axial speed distribution.

$\overline{\varphi}_i$ = **phi**, the mean molecular impulse in newtons.

$\bar{\rho}$ = **rho**, the mean atmospheric specific mass density in kilograms per cubic meter for humid air.

$\bar{\rho}_{\sim}$ = **rho**, the mean atmospheric specific mass density in kilograms per cubic meter for water vapor.

$\bar{\rho}_{\cdot}$ = **rho**, the mean atmospheric specific mass density in kilograms per cubic meter for dry air.

$\bar{\rho}_{\rightarrow}$ = mean molecular momentum normal to the surface for humid air

$\bar{\rho}_{\underset{\sim}{\rightarrow}}$ = mean molecular momentum normal to the surface for water vapor

$\bar{\rho}_{\underset{\cdot}{\rightarrow}}$ = mean molecular momentum normal to the surface for dry air

$\bar{\lambda}_i$ = mean free molecular path normal to the surface in meters

$\bar{\lambda}_{i'}$ = mean free molecular true path in meters

$\underset{\cdot}{X}$ = a dot under the parameter symbol means that that parameter is characteristic only of dry air

$\underset{\sim}{X}$ = a tilde under the parameter symbol means that that parameter is characteristic only of water vapor

X = no symbol under the parameter symbol means that that parameter is characteristic of humid air

REFERENCES

Click the link in the Discussion line and see the references in the defining essay on that topic.